Vortices in multicomponent exciton-polariton superfluids

Yura Rubo

Universidad Nacional Autónoma de México, México

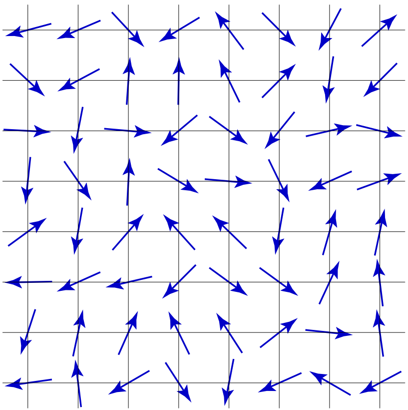
Outline:

- XY-model. BKT transition. Superfluids
- Spin-1 condensates. The Berry phase. Half-vortices
- Spin-2 condensates. One-third vortices
- Condensation of exciton-polaritons in microcavities
- Half-vortices in exciton-polariton condensates
- Warping of vortices in the presence of TE-TM splitting
- Interactions and peculiarities of BKT transition
- Observations of half-vortices and polarization vortices

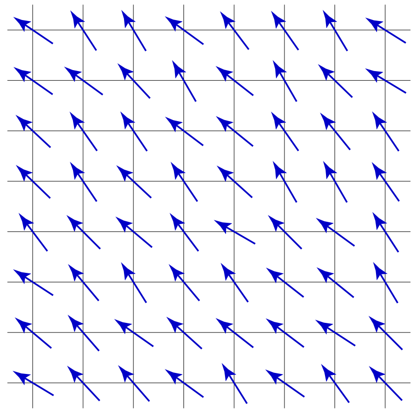
Phase-transition in XY-model

The symmetry is lowered due to the disorder-order transition:

$T > T_c$



$T < T_c$

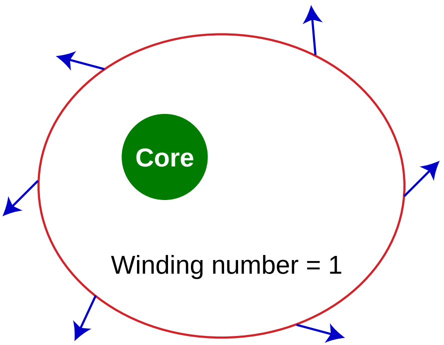


$$H = \rho_s \sum_{\langle ij \rangle} [1 - \cos(\theta_i - \theta_j)] = \frac{1}{2}\rho_s \sum_{\langle ij \rangle} [\theta(\vec{r}_i) - \theta(\vec{r}_j)]^2$$

$$= \frac{1}{2}\rho_s \sum_i a^2 |\nabla\theta(\vec{r}_i)|^2 = \frac{1}{2}\rho_s \int d^2r |\nabla\theta(\vec{r})|^2.$$

XY-model: Vortices and winding numbers

Apart from spin waves, the vortices are important (V. L. Berezinskii, 1970). In vortex, there is rotation of the spin on distances $\gg a$ (the core size), with a resulting change multiple by 2π:



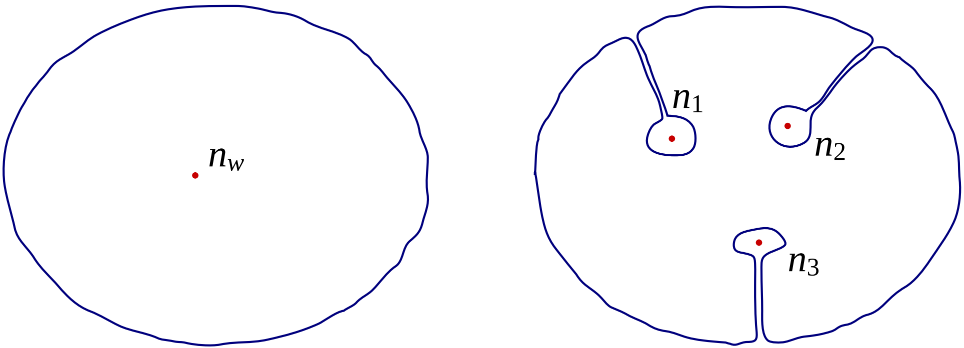
Variation of $\theta(\vec{r})$ is subject to minimization of the Hamiltonian energy,

$$-\rho_s \Delta\theta(\vec{r}) = 0, \qquad \theta = \theta_0 + n_w \phi.$$

The winding number $n_w = 0, \pm 1, \pm 2, \ldots$. Mathematically, $\pi_1(S_1) = Z$.

Topological invariance and topological stability

$$n_w = n_1 + n_2 + n_3$$



Single vortex is topologically stable. It cannot be transformed into the ground state ($n_w = 0$).

A $n_1 = +1$ and $n_2 = -1$ vortex pair can be created from and transformed into the ground state, since $n_1 + n_2 = 0$.

The Berezinskii-Kosterlitz-Thouless (BKT) transition

The single vortex energy

$$E_s = \frac{1}{2}\rho_s \int d^2r |\nabla\theta|^2 = \frac{1}{2}\rho_s \int d^2r \left(\frac{1}{r}\frac{d\theta}{d\phi}\right)^2 = \pi\rho_s k^2 \int_a^R \frac{1}{r} dr = \pi\rho_s n_w^2 \ln\left(\frac{R}{a}\right).$$

The energy of a pair of vortices is finite for $n_1 + n_2 = 0$,

$$E_p = \pi\rho_s(n_1+n_2)^2 \ln(R/a) - 2\pi\rho_s n_1 n_2 \ln(r/a).$$

The critical temperature can be found by
[J. M. Kosterlitz and D. J. Thouless, (1973); J. M. Kosterlitz, (1974)]

$$F = E_s - TS = \pi\rho_s \ln(R/a) - T\ln(R^2/a^2) = (\pi\rho_s - 2T)\ln(R/a),$$

so that single vortices appear and destroy the order at

$$T_c = \frac{\pi}{2}\rho_s.$$

Bose-Einstein condensation (BEC)

Ideal Bose gas. In the 3D case:

$$N = N_0 + \frac{V}{(2\pi)^3}\int_0^\infty \frac{4\pi k^2 \mathrm{d}k}{\exp\{\frac{\omega_0(k)-\mu}{T}\}-1} = N_0 + N_{\mathrm{ex}}(\mu, T), \qquad \omega_0(k) = \frac{k^2}{2m^*}.$$

Condensation temperature T_{BE} is the root of $N_{\mathrm{ex}}(0, T_{\mathrm{BE}}) = N$, so that
$\mu < 0$ for $T > T_{\mathrm{BE}}$. One can omit N_0.
$\mu = 0$ for $T < T_{\mathrm{BE}}$ and $N_0 \propto V$.
There is macroscopic number of bosons in the state with $\vec{k} = 0$.
The order parameter $\psi = \sqrt{n}\mathrm{e}^{\mathrm{i}\theta}$, where $n = N_0/V$.

The result is different in the 2D case:

$$N_{\mathrm{ex}}(\mu, T) = \frac{A}{(2\pi)^2}\int_0^\infty \frac{2\pi k \mathrm{d}k}{\exp\{\frac{\omega_0(k)-\mu}{T}\}-1} \qquad \text{is divergent at } \mu = 0,\ T > 0.$$

so that $\mu < 0$ for all $T > 0$.
There is no BEC (as a phase transition) in 2D at finite temperatures.
This is a particular case of the Mermin-Wagner theorem about the absence of true long-range order in 2D at $T > 0$.

Effects of interactions. Superfluidity

Repulsive interaction $U_0 \equiv U(k=0) > 0$ changes qualitatively the quasi-particle spectrum. One has $\mu = nU_0 > 0$ and (N. N. Bogoliubov, 1947):

$$\omega^2(k) = \omega_0^2(k) + 2nU_0\omega_0(k) = \omega_0^2(k) + 2\mu\omega_0(k), \qquad \omega_0 = k^2/2m^*,$$

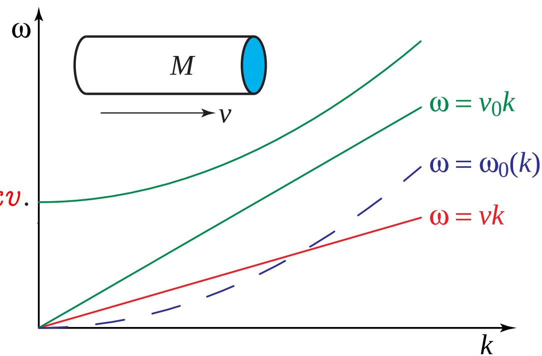
so that $\omega = v_0 k$ with the sound velocity $v_0 = \sqrt{nU_0/m^*}$ for small k.

This results in superfluidity (L. D. Landau, 1940). One needs

$$Mv = P = P' + k,$$

$$\frac{P^2}{2M} = \frac{(P-k)^2}{2M} + \omega(k), \text{ or } \omega(k) = kv.$$

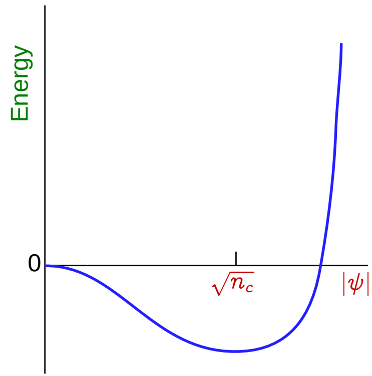
The flow with $v < \omega(k)/k$ is ideal. No losses and viscosity.



In 2D case, a weak repulsive interaction makes possible a superfluid transition with $T_c > 0$.

Mean-field theory

$$H = \int d^d r \left\{ \frac{\hbar^2}{2m^*} |\vec{\nabla}\psi(r)|^2 - \mu|\psi|^2 + \frac{1}{2} U_0 |\psi|^4 \right\}.$$



$$T \to 0: \ \mu = U_0 n, \quad \psi = \sqrt{n_c}\, e^{i\theta}$$

Considering vortices one can assume $|\psi|^2 = n_c$ (otherwise *energy* $\propto$ *area*), so $\psi = \sqrt{n_c}\, e^{i\theta(\vec{r})}$, and

$$H_{el} = \frac{1}{2}\rho_s \int d^d r \, |\nabla\theta(\vec{r})|^2, \quad \rho_s = \frac{\hbar^2 n_c}{2m^*}.$$

The vortex core. Condition $|\psi| = \sqrt{n_c}$ is violated on distances $r \lesssim a = \hbar/\sqrt{2m^*\mu}$, that define the vortex core.

Singularity at $r \to 0$, where $\psi \propto r e^{\pm i\phi}, \ r^2 e^{\pm 2i\phi}, \ldots$ for $n_w = \pm 1, \ \pm 2, \ldots$.

Multicomponent superfluids. Mixing of the components

The order parameter $\psi = (\psi_F, \psi_{F-1}, \ldots, \psi_m, \ldots, \psi_{-F})^T$.

Simple classification: vortex in one component, with the other component being regular. If $\psi_i = |\psi_i| e^{i\theta_i}$, one can have

$$\theta_m \to \theta_m + n_w \phi, \qquad \theta_{m'} \to \theta_{m'} \ \ (\text{for } m' \neq m).$$

In general, this classification is not good: there is mixing of components, and the phases are not independent.

Mixing due scattering. E.g., of two $m = 0$ atoms into $m = +1$ and $m = -1$:

$$H_{mix} = \frac{1}{2} V_{mix} (\psi_{+1}^* \psi_{-1}^* \psi_0 \psi_0 + \psi_0^* \psi_0^* \psi_{+1} \psi_{-1}) \\ = V_{mix} |\psi_{+1}||\psi_{-1}||\psi_0|^2 \cos(\theta_{+1} + \theta_{-1} - 2\theta_0).$$

So that the change of phases is subject to the constriction $\theta_{+1} + \theta_{-1} - 2\theta_0 = n\pi$.

The other source of mixing is possible dependence of the mass of the particle on the direction of ψ (like longitudinal-transverse splitting).

Spin=1 BEC: Ferromagnetic and Polar Phases

For $\psi = (\psi_{+1}, \psi_0, \psi_{-1})^T$ the interaction energy is

$$H_{int} \propto V_0 n^2 + V_1 |\vec{F}|^2, \qquad n = \psi^\dagger \cdot \psi, \qquad \vec{F} = \psi^\dagger \cdot \vec{f} \cdot \psi,$$

$$f_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad f_y = \frac{\mathrm{i}}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad f_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Rotation operator $U(\alpha, \beta, \gamma) = e^{-\mathrm{i} f_z \alpha} e^{-\mathrm{i} f_y \beta} e^{-\mathrm{i} f_z \gamma}$.

Ferromagnetic phase

$V_1 < 0$

$$\psi = \sqrt{n} e^{\mathrm{i}\theta} U(\alpha, \beta, 0) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \sqrt{\frac{n}{2}}\, e^{\mathrm{i}\theta} \begin{pmatrix} \sqrt{2} e^{-\mathrm{i}\alpha} \cos^2(\beta/2) \\ \sin(\beta) \\ \sqrt{2} e^{\mathrm{i}\alpha} \sin^2(\beta/2) \end{pmatrix}.$$

Polar phase

$V_1 > 0$

$$\psi = \sqrt{n} e^{\mathrm{i}\theta} U(\alpha, \beta, 0) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= \sqrt{\frac{n}{2}}\, e^{\mathrm{i}\theta} \begin{pmatrix} -e^{-\mathrm{i}\alpha} \sin(\beta) \\ \sqrt{2} \cos(\beta) \\ e^{\mathrm{i}\alpha} \sin(\beta) \end{pmatrix}.$$

Spin=1 Ferromagnetic: The Berry phase

$\hat{s} = \vec{F}/n = (\sin\beta\cos\alpha, \sin\beta\sin\alpha, \cos\beta)$

The mass velocity

$$\vec{v} = \frac{i\hbar}{2m^* n}\sum_m (\psi_m(\vec{\nabla}\psi_m^*) - \psi_m^*(\vec{\nabla}\psi_m)) = \frac{\hbar}{m^*}(\vec{\nabla}\theta - \cos\beta\vec{\nabla}\alpha).$$

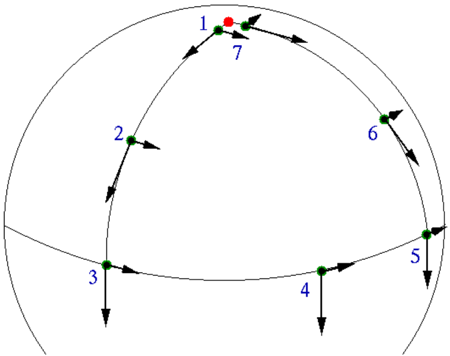
The Mermin-Ho relation:

$\vec{\nabla}\times\vec{v} = (\hbar/2m^*)\sin\beta[\vec{\nabla}\beta\times\vec{\nabla}\alpha] = (\hbar/2m^*)\epsilon_{\mu\nu\lambda}\hat{s}_\mu[\vec{\nabla}\hat{s}_\nu\times\vec{\nabla}\hat{s}_\lambda] \neq 0.$

$$\vec{v} - \frac{\hbar}{m^*}(1-\cos\beta)\vec{\nabla}\alpha = \frac{\hbar}{m^*}\vec{\nabla}(\theta-\alpha)$$

$$\oint \vec{v}\cdot d\vec{l} - \frac{\hbar}{m^*}\oint(1-\cos\beta)\vec{\nabla}\alpha\cdot d\vec{l}$$

$$= \frac{h}{m^*}n_w, \text{ with } n_w \in Z.$$



Spin=1 Ferromagnetic: Topological stability and vortex creation.

Consider $\alpha = n_w\phi$ and $\theta = -n_w\phi$:

$$\psi = \sqrt{n}\left(\cos^2\frac{\beta}{2}, \frac{e^{in_w\phi}}{\sqrt{2}}\sin\beta, e^{2in_w\phi}\sin^2\frac{\beta}{2}\right)^T = \sqrt{n}\begin{cases}(1,0,0) \text{ for } \beta = 0,\\ (0,0,e^{2in_w\phi}) \text{ for } \beta = \pi.\end{cases}$$

The vortex in ± 1-component with an even winding number is unstable.
It can be used to create such a vortex form the ground state by applying adiabatic magnetic field (W. Ketterle group, 2002):

$$\vec{B} = (B_\perp\cos(-\phi), B_\perp\sin(-\phi), B_z)^T,$$

that gives $\alpha = \phi$ and $\beta = \arctan(B_\perp/B_z)$.

Also,

$$(0,0,e^{i(2n_w+1)\phi})^T \leftrightarrow (e^{i\phi},0,0)^T.$$

Spin=1 Ferromagnetic: Two types of vortices

The coreless vortex. $\theta = \alpha = \pm\phi, \quad \beta(r=0) = 0, \quad \beta(r=r_0) = \pi,$

$$\psi_{cl} = \sqrt{n} \begin{pmatrix} \cos^2(\beta/2) \\ \frac{e^{i\phi}}{\sqrt{2}} \sin\beta \\ e^{2i\phi} \sin^2(\beta/2) \end{pmatrix}, \qquad \vec{v} = \frac{\hbar}{m^* r}(1 - \cos\beta)\hat{\phi}.$$

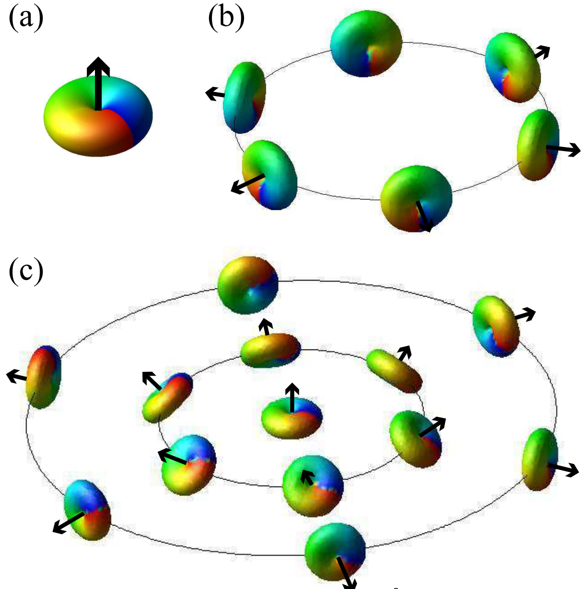
The polar-core vortex. $\theta = 0, \quad \alpha = \pm\phi, \quad \beta = const,$

$$\psi_{pl} = \sqrt{n} \begin{pmatrix} e^{-i\phi} f(r) \cos^2(\beta/2) \\ \left[1 - f^2(r)\cos^4(\beta/2) - g^2(r)\sin^4(\beta/2)\right]^{1/2} \\ e^{i\phi} g(r) \sin^2(\beta/2) \end{pmatrix},$$

where $f(0) = g(0) = 0, \quad f(\infty) = g(\infty) = 1.$

Spin=1 Ferromagnetic: Two types of vortices

Profile of the order parameter: $\sum_m \psi_m Y_{1m}(\hat{s})$



[From Ueda and Kawaguchi, (2010)]

Spin=1 Polar: Half-vortices

Spin quantization axis $\hat{d} = (\sin\beta\cos\alpha, \sin\beta\sin\alpha, \cos\beta)$

$$\psi_{pol} = \sqrt{\frac{n}{2}}\, e^{\mathrm{i}\theta} \begin{pmatrix} -e^{-\mathrm{i}\alpha}\sin\beta \\ \sqrt{2}\cos\beta \\ e^{\mathrm{i}\alpha}\sin\beta \end{pmatrix}, \qquad (\hat{d}\cdot\vec{f})\psi_{pol} = 0, \qquad \vec{v} = \frac{\hbar}{m^*}\vec{\nabla}\theta.$$

While there is no Berry phase in quantization of supercurrent, the spin-gauge symmetry leads to existence of half-quantum vortices:

$$\psi \to \psi, \text{ when } \hat{d} \to -\hat{d}, \quad \theta \to \theta \pm \pi.$$

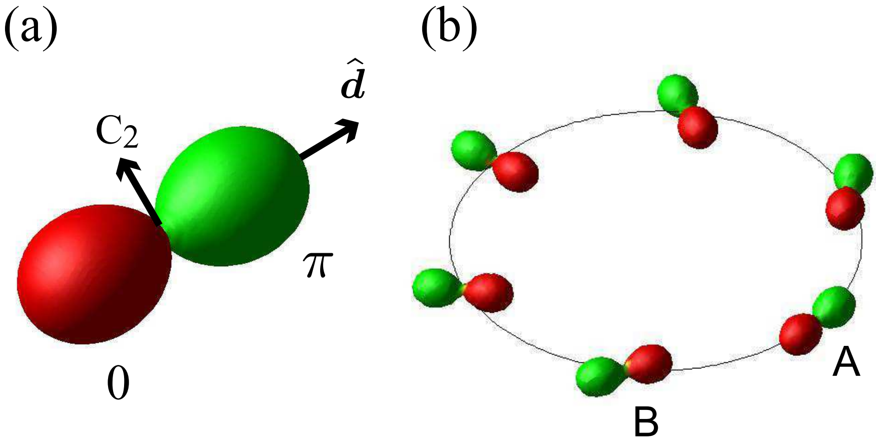
For example, setting $\beta = \pi/2$, $\alpha = n_w\phi/2$, $\theta = n_w\phi/2$, we have

$$\psi = \sqrt{\frac{n}{2}} \begin{pmatrix} -1 \\ 0 \\ e^{\mathrm{i}n_w\phi} \end{pmatrix}, \qquad \oint \vec{v}\cdot d\vec{l} = \frac{h}{2m^*} n_w.$$

In 3D the half-vortex line is also referred to as the Alice string.

Spin=1 Polar: Half-vortices

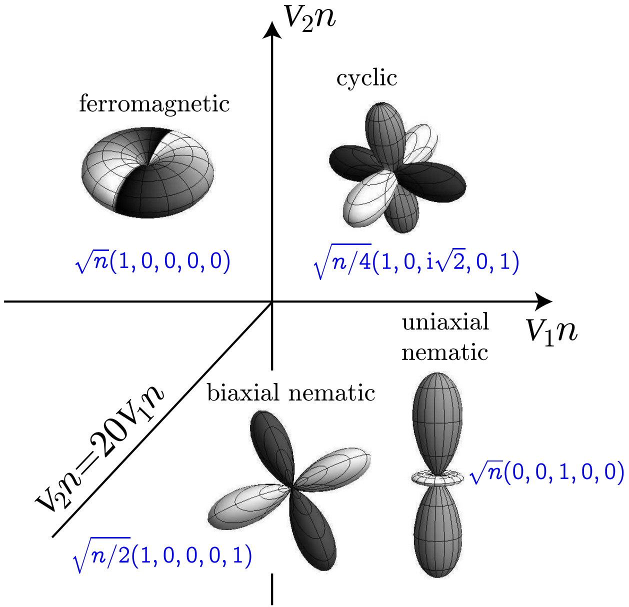
The order parameter defines the surface $|\sum_m \psi_m Y_{1m}(\hat{s})|^2$.
Colors indicate combined spin-gauge symmetry.



[From Ueda and Kawaguchi, (2010)]

Phases of Spin=2 Condensates

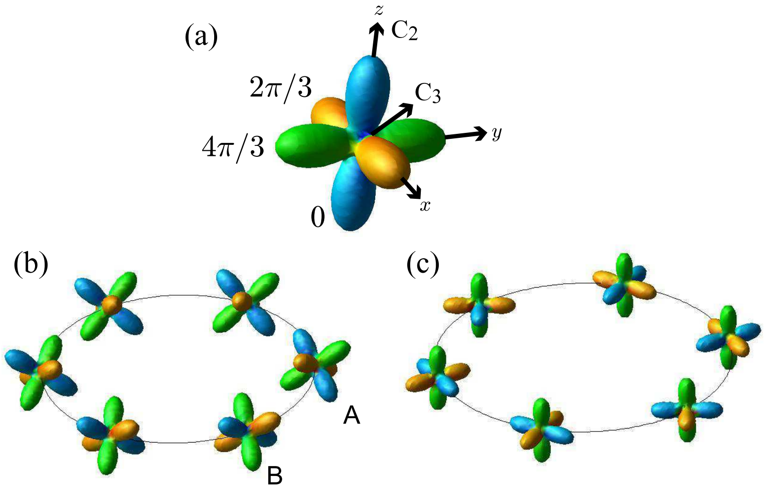
$$H_{int} \propto V_0 n^2 + V_1|\vec{F}|^2 + V_2|A|^2, \quad A = [2\psi_{-2}\psi_2 - 2\psi_1\psi_1 + \psi_0^2]/\sqrt{5}.$$



One-third vortices in the cyclic phase

Cyclic state $\xi_{cyc} = (1, 0, \mathrm{i}\sqrt{2}, 0, 1)^T$. For $\varphi = 2\pi/3$

$$\exp\left\{-\mathrm{i}\frac{f_x + f_y + f_z}{\sqrt{3}}\varphi\right\} e^{\mathrm{i}\theta}\xi_{cyc} = e^{4\pi\mathrm{i}/3} e^{\mathrm{i}\theta}\xi_{cyc}, \qquad \theta \to \theta + \frac{2\pi}{3}.$$

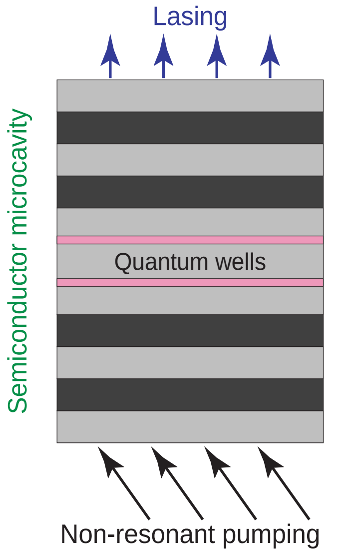


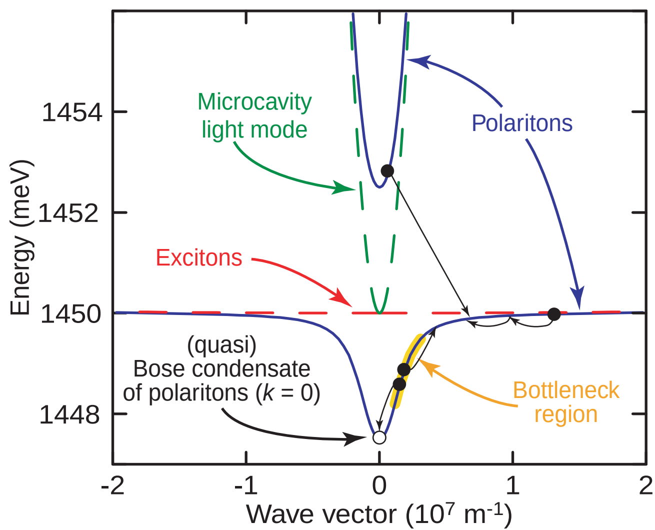
[From Ueda and Kawaguchi, (2010)]

References

- P.M. Chaikin, T.C. Lubensky, *Principles of Condensed Matter Physics*, Cambridge University Press, Cambridge, (1995).
- G. E. Volovik, *The Universe in a Helium Droplet*, Oxford University Press, New York, (2003).
- M. Ueda, Y. Kawaguchi, *Spinor Bose-Einstein condensates*, arXiv:1001.2072.

Polariton condensation and lasing

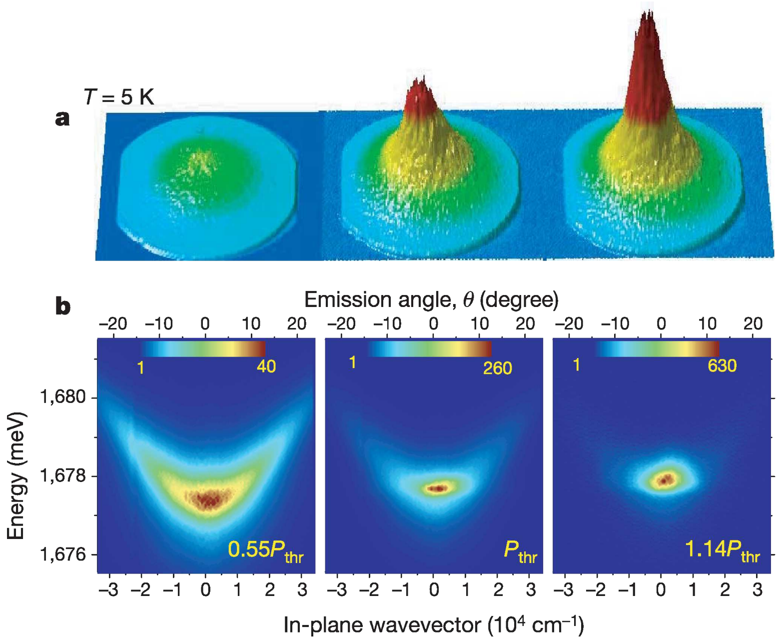




- Effective mass of lower branch polaritons: $m^* \sim 10^{-4} m_0$.
- Relaxation problem: bottleneck for $d\omega/dk > v_{\text{sound}}$.
- Presence of polarization degree of freedom.
  The order parameter is complex 2D vector $\vec{\psi} \propto \vec{\mathcal{E}}_{\parallel}$.

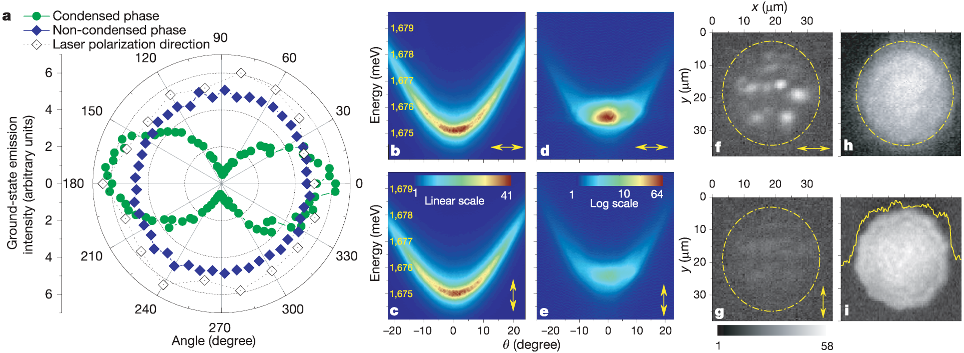
First observation of condensation (Le Si Dang group)

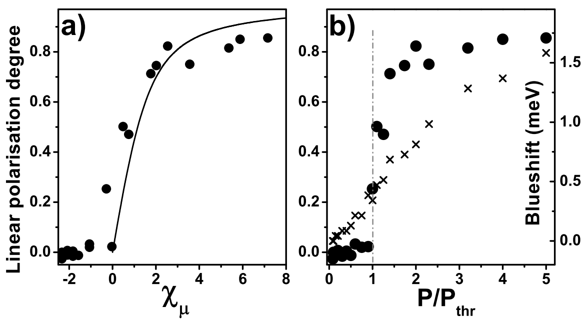
Emission from the CdTe-based microcavity:



From J. Kasprzak *et al.*, Nature 443, 409 (2006).

Spontaneous polarization formation (Le Si Dang group)





From J. Kasprzak *et al.*, Nature 443, 409 (2006) and Phys. Rev. B 75, 045326 (2007).

Linear polarization of the condensate

Formation of linear polarization in polariton condensates [Le Si Dang *et al.*; Snoke *et al.*, 2006] arises due to the reduction of polariton-polariton repulsion energy $H_{\rm int}$:

$$H_{\rm int} = \frac{1}{2}\int d^2r \left\{ (U_0 - U_1)(\vec{\psi}^* \cdot \vec{\psi})^2 + U_1 |\vec{\psi}^* \times \vec{\psi}|^2 \right\}.$$

Two interaction constants, $U_0 = AM_{\uparrow\uparrow}$ and $U_1 = A(M_{\uparrow\uparrow} - M_{\uparrow\downarrow})/2$, where $A = \pi R^2$ is the excitation spot area. Typically, $U_0/2 < U_1 < U_0$. At a fixed concentration $n = (\vec{\psi}^* \cdot \vec{\psi})$ minimum of $H_{\rm int}$ is reached for

$$\vec{\psi}^* \times \vec{\psi} = 0 \;\Rightarrow \text{Linear polarization}$$

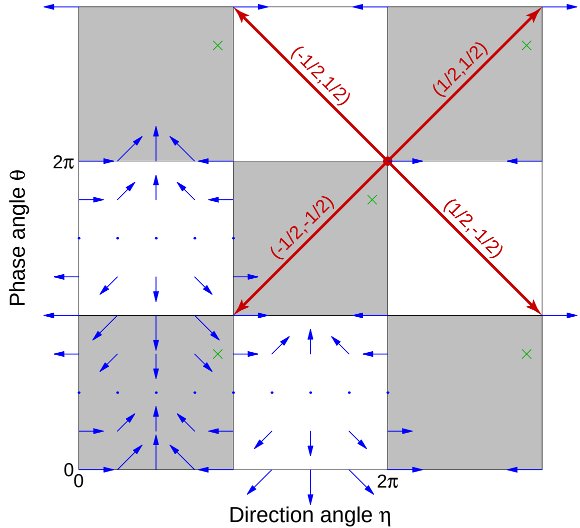
One can write $\quad \vec{\psi}_{\rm lin} = \{\psi_x, \psi_y\} = \sqrt{n}\, e^{i\theta}\{\cos\eta, \sin\eta\},$

so that the order parameter is defined by two angles, η and θ.

The states η, θ and $\eta + \pi, \theta + \pi$ are identical.
The order parameter manifold $M = (U(1) \times S_1)/\mathbb{Z}_2$.
The first homotopy group $\pi_1(M) = \mathbb{Z} \times \mathbb{Z}$.

The order parameter space

$$\vec{\psi}_{\rm lin} = \{\psi_x, \psi_y\} = \sqrt{n}\, e^{i\theta}\, \{\cos\eta, \sin\eta\}\,.$$



The possible changes are:
$\eta \to \eta + 2\pi k$,
$\theta \to \theta + 2\pi m$.

Vortex carries two topological charges (winding numbers), (k, m).

Integer vortices:
$k, m = 0, \pm 1, \pm 2, \ldots$

Half-integer vortices:
$k, m = \pm 1/2, \pm 3/2, \ldots$

Half-vortices

Half-vortices in ^{3}He-A: G.E. Volovik and V.P. Mineev (1976); M.C. Cross and W.F. Brinkman (1977).

They appear due to combined spin-gauge symmetry:

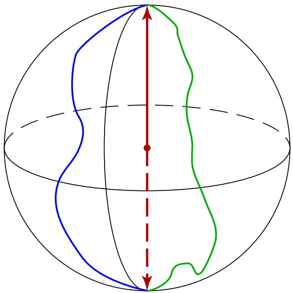
Spin quantization axis change $\vec{d} \to -\vec{d}$

Phase change $\theta \to \theta \pm \pi$

The superfluid velocity around the half-vortex $\vec{v}_s \propto \nabla\theta$ is a half of the superfluid velocity around the usual vortex with $\theta \to \theta \pm 2\pi$.

Half-vortex carries half-quantum of the superfluid current.

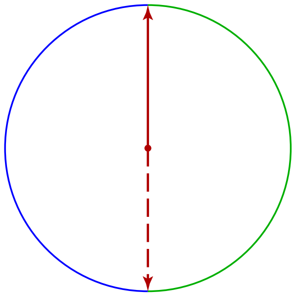
Why two winding numbers (k, m)?



Atomic spin-1 condensates
(three-component)

3D real $\vec{d}$ and phase θ
Half-vortex: $\vec{d} \to -\vec{d}$, $\theta \to \theta + \pi$

All rotations $\vec{d} \to -\vec{d}$ can be transformed to each other



Polariton pseudospin case
(two-component)

2D real $\vec{d}$ and phase θ
Half-vortex: $\vec{d} \to -\vec{d}$, $\theta \to \theta + \pi$

Clockwise and counterclockwise $\vec{d} \to -\vec{d}$ are topologically distinct

Half-vortex in the circular polarization basis

The circular components are defined by

$$\vec{\psi} = \hat{\vec{x}}\psi_x + \hat{\vec{y}}\psi_y = \frac{\hat{\vec{x}} + \mathrm{i}\,\hat{\vec{y}}}{\sqrt{2}}\psi_{+1} + \frac{\hat{\vec{x}} - \mathrm{i}\,\hat{\vec{y}}}{\sqrt{2}}\psi_{-1}.$$

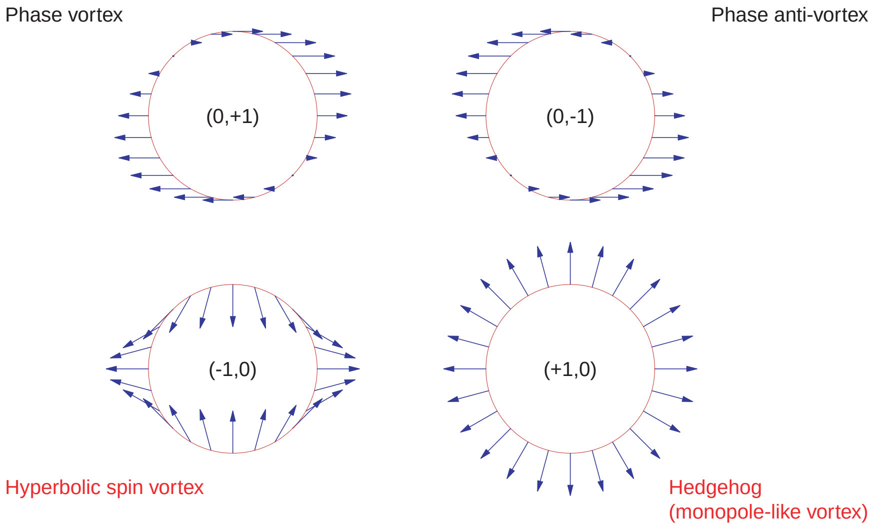
Consider $\eta = k\phi$ and $\theta = m\phi$, where ϕ is the azimuthal angle, then

$$\psi_{+1} = \sqrt{\frac{n}{2}}\, e^{\mathrm{i}(m-k)\phi}, \qquad \psi_{-1} = \sqrt{\frac{n}{2}}\, e^{\mathrm{i}(m+k)\phi}.$$

Right half-vortices: $k = m$. Left-circular component becomes fully depleted and polarization is right-circular at $r = 0$.

Left half-vortices: $k = -m$. Right-circular component becomes fully depleted and polarization is left-circular at $r = 0$.

Integer vortices



$$(0,+1) \rightarrow (+\tfrac{1}{2},+\tfrac{1}{2}) + (-\tfrac{1}{2},+\tfrac{1}{2}), \qquad (0,-1) \rightarrow (+\tfrac{1}{2},-\tfrac{1}{2}) + (-\tfrac{1}{2},-\tfrac{1}{2}),$$

$$(-1,0) \rightarrow (-\tfrac{1}{2},+\tfrac{1}{2}) + (-\tfrac{1}{2},-\tfrac{1}{2}), \qquad (+1,0) \rightarrow (+\tfrac{1}{2},+\tfrac{1}{2}) + (+\tfrac{1}{2},-\tfrac{1}{2}).$$

The hedgehog $(+1,0)$ restores the polarization S_1 symmetry (C_∞).

The elastic energy for $m_t = m_l = m^*$

The elastic energy in the case when polariton mass does not depend on polarization

$$H_{\rm el} = \frac{1}{2}\rho_s \int d^2r \left[(\nabla\eta)^2 + (\nabla\theta)^2\right], \qquad \rho_s = \frac{\hbar^2 n}{m^*}.$$

The elastic field of half-vortices is simple: $\Delta\theta = 0$ and $\Delta\eta = 0$, so that $\theta = \pm\frac{1}{2}\phi$ and $\eta = \pm\frac{1}{2}\phi$. Logarithmic prefactor and $T_{\rm KT}$:

$$E_s = \frac{1}{2}\pi\rho_s, \qquad T_{\rm KT} = \frac{1}{2}E_s = \frac{1}{4}\pi\rho_s.$$

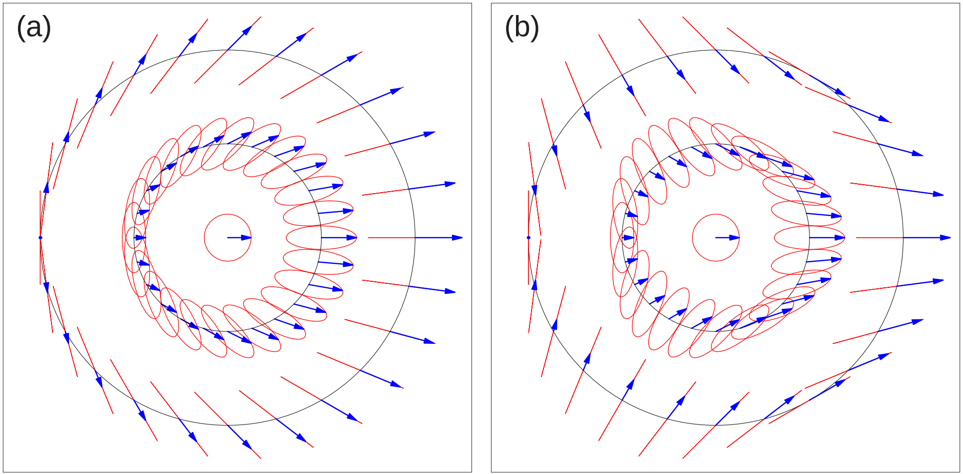
The other way is to introduce the phases of circular components, $\theta_+ = \theta - \eta$ and $\theta_- = \theta + \eta$, then

$$H_{\rm el} = \frac{1}{4}\rho_s \int d^2r \left[(\nabla\theta_+)^2 + (\nabla\theta_-)^2\right],$$

and the half-vortex is a full vortex in only one circular component.

The polarization texture of half-vortex core

Showing $\mathrm{Re}\{\vec{\psi} e^{-\mathrm{i}\omega t}\}$, where $\omega = \omega_p + \mu$.



"Lemon" "Star"

Two left half-vortices

Pair of left half-vortices

Permuted pair of left half-vortices

The Gross-Pitaevskii equation

The Gross-Pitaevskii equation is

$$i\hbar\frac{\partial}{\partial t}\vec{\psi}(\vec{r},t) = \frac{\delta H}{\delta\vec{\psi}^*(\vec{r},t)}, \qquad H = \int \mathcal{H}(\vec{\psi}^*,\vec{\psi})d^2r,$$

The energy density for the polariton superfluid:

$$\mathcal{H} = \mathcal{T} - \mu n + \mathcal{H}_{\text{int}} + \mathcal{H}'.$$

Spin-dependence of the kinetic energy and interactions

$$\mathcal{T} = \frac{\hbar^2}{2m_l}|\vec{\nabla}\cdot\vec{\psi}|^2 + \frac{\hbar^2}{2m_t}|\vec{\nabla}\times\vec{\psi}|^2,$$

$$\mathcal{H}_{\text{int}} = \frac{1}{2}(U_0 - U_1)(\vec{\psi}^*\cdot\vec{\psi})^2 + \frac{1}{2}U_1|\vec{\psi}^*\times\vec{\psi}|^2.$$

m_l is longitudinal or transverse-magnetic (TM) mass,
m_t is transverse or transverse-electric (TE) mass.

TE-TM splitting "problem"

Consider only the kinetic energy terms (in circular basis)

$$\mathrm{i}\dot{\psi}_{+1} = -\frac{1}{2m^*}\left[\Delta\psi_{+1} + 4\gamma\frac{\partial^2}{\partial z^2}\psi_{-1}\right] + \ldots,$$

$$\mathrm{i}\dot{\psi}_{-1} = -\frac{1}{2m^*}\left[\Delta\psi_{-1} + 4\gamma\frac{\partial^2}{\partial z^{*2}}\psi_{+1}\right] + \ldots,$$

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - \mathrm{i}\frac{\partial}{\partial y}\right), \quad \frac{1}{m^*} = \frac{1}{2}\left(\frac{1}{m_l} + \frac{1}{m_t}\right), \quad \gamma = \frac{m_t - m_l}{m_t + m_l}.$$

There are ± 2 moment transfer between left and right components.

So, one cannot have solution like $\psi_{+1} \propto e^{\mathrm{i}\phi}$ and $\psi_{-1} \propto \mathrm{const}(\phi)$, because $(\partial^2/\partial z^2)\psi_{-} \propto e^{-2\mathrm{i}\phi}$.

Do half-vortices exist?

Elastic energy in presence of TE-TM splitting

In-plane electric field defines $\hat{\vec{n}} = \{\cos\eta, \sin\eta\}$. The elastic energy is

$$H_{\rm el} = \frac{1}{2}\int d^2r \left(\rho_l \left\{(\hat{\vec{n}}\cdot\vec{\nabla}\theta)^2 + [\hat{\vec{n}}\times\vec{\nabla}\eta]^2\right\} + \rho_t \left\{(\hat{\vec{n}}\cdot\vec{\nabla}\eta)^2 + [\hat{\vec{n}}\times\vec{\nabla}\theta]^2\right\}\right),$$

where $\rho_l = \hbar^2 n/m_l$ and $\rho_t = \hbar^2 n/m_t$.

Nonlinear field equations. Not like $\Delta\theta = 0$ and $\Delta\eta = 0$, with solutions $\theta \propto \phi$ and $\eta \propto \phi$ as before.

One needs to find the correct boundary conditions, i.e., $\theta(\phi)$ and $\eta(\phi)$ for large distances.

Asymptotic behavior

At large distances:

$$\psi_{\pm 1}(r \gg a, \phi) = \sqrt{\frac{n}{2}}\, e^{\mathrm{i}[\theta(\phi) \mp \eta(\phi)]}.$$

Vortex minimizes the energy for specific topological sector

$$\eta(\phi + 2\pi) - \eta(\phi) = 2\pi k, \qquad \theta(\phi + 2\pi) - \theta(\phi) = 2\pi m.$$

The vortex energy is $E_{\mathrm{vor}} = E_c + E_s \ln(R/a)$ and

$$E_s = \frac{\hbar^2 n}{2m^*} \int_0^{2\pi} \left\{ [1 + \gamma \cos(2u)](1 + u')^2 + [1 - \gamma \cos(2u)]\theta'^2 \right\} d\phi,$$

where the prime denotes the derivative over ϕ and

$$u(\phi) = \eta(\phi) - \phi.$$

Boundary conditions

By variation we obtain

$$[1-\gamma\cos(2u)]\,\theta'' + 2\gamma\sin(2u)u'\theta' = 0,$$

$$[1+\gamma\cos(2u)]\,u'' + \gamma\sin(2u)\left(1 - u'^2 - \theta'^2\right) = 0,$$

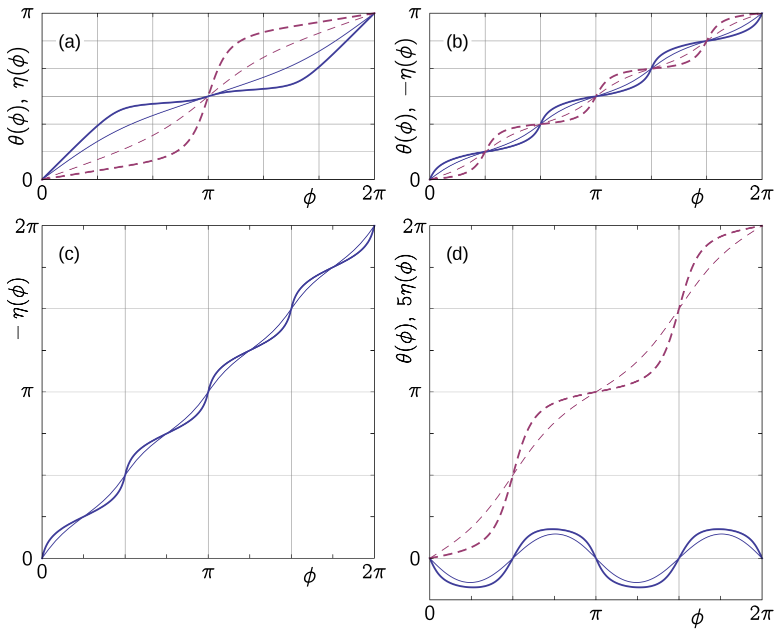
$$u(0)=0, \quad \theta(0)=0, \quad u(2\pi)=2(k-1)\pi, \quad \theta(2\pi)=2m\pi.$$

Simple particular vortices,

(i) *Hedgehog vortices.* These are $(1,m)$-vortices having $\theta = m\phi$ and $u \equiv 0$, so that $\eta = \phi$. In particular, monopole-like solution $(1,0)$.

(ii) *Double-quantized polarization vortex* $(2,0)$. In this special case $\theta \equiv 0$, but $u = \phi$, resulting in $\eta = 2\phi$.

Nonlinear behavior of angles



$\gamma = -0.4$ (thin lines) and $\gamma = -0.9$ (thick lines).
The $(\frac{1}{2}, \frac{1}{2})$ half-vortex (a), the $(-\frac{1}{2}, \frac{1}{2})$ half-vortex (b).
The $(-1, 0)$ hyperbolic polarization vortex (c), and the $(0, 1)$ phase vortex (d).

Nonlinearity of angles (qualitatively)

Using the effective masses for the phase m_θ and for the polarization m_η,

$$\frac{1}{m_\theta} = \frac{\cos^2 u}{m_t} + \frac{\sin^2 u}{m_l}, \qquad \frac{1}{m_\eta} = \frac{\sin^2 u}{m_t} + \frac{\cos^2 u}{m_l},$$

where $u(\phi) = \eta(\phi) - \phi$. The energy prefactor functional is

$$E_s = \frac{\hbar^2 n}{2} \int_0^{2\pi} \left\{ \frac{\eta'^2}{m_\eta} + \frac{\theta'^2}{m_\theta} \right\} d\phi.$$

There are sectors where $m_\theta \approx m_l$ and $m_\eta \approx m_t$, and there are sectors where $m_\theta \approx m_t$ and $m_\eta \approx m_l$.

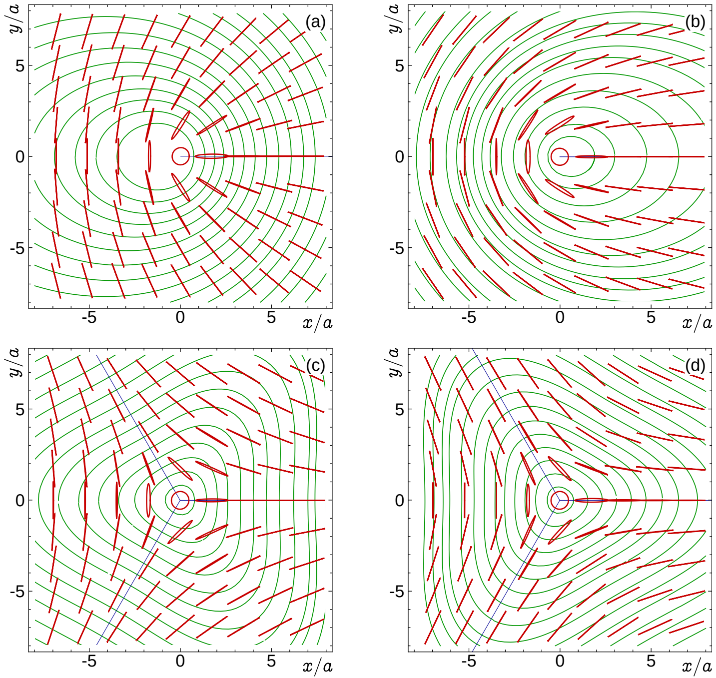
The main change of θ and η happens in the sectors where the “angle” mass is heavy.

Warped half-vortices (nimerical solutions)

Lemons, $k = +\frac{1}{2}$

(*Monstars* are not realized)

Stars, $k = -\frac{1}{2}$



$\gamma = -0.5$ $\gamma = 0.5$

Long-range half-vortex interactions

Without TE-TM splitting.

Right and left half-vortices do not interact.

Independent proliferation of $(+\frac{1}{2}, +\frac{1}{2})-(-\frac{1}{2}, -\frac{1}{2})$ and $(\frac{1}{2}, -\frac{1}{2})-(-\frac{1}{2}, +\frac{1}{2})$.

Interaction of within each pair is $V(r) = (1/2)E_0 \ln(r/a)$.

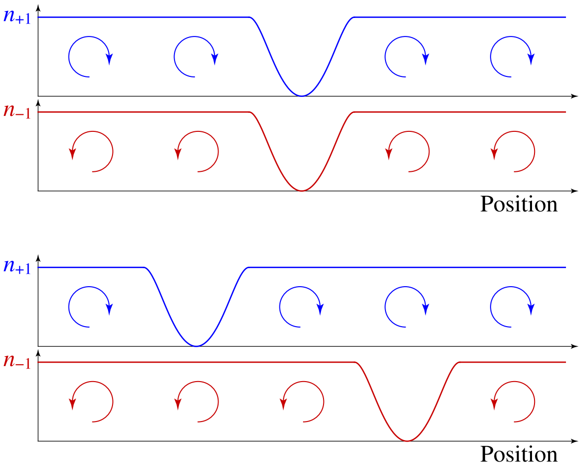
Two decoupled BKT transitions with $T_c = (1/4)E_0$, where $E_0 = \pi\hbar^2 n/m^*$.

With TE-TM splitting, $\gamma = (m_t - m_l)/(m_t + m_l)$.

The interactions between left and right half-vortices:

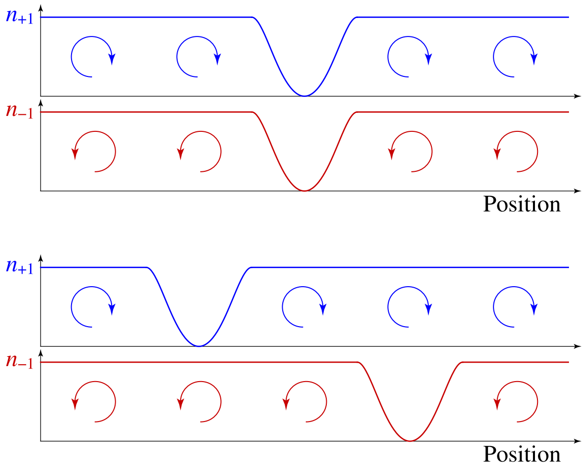
Pair	**Interaction potential**
$(+\frac{1}{2}, +\frac{1}{2})$ and $(+\frac{1}{2}, -\frac{1}{2})$	$-\gamma E_0 \ln(r/a)$
$(-\frac{1}{2}, +\frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$	$-(7/32)\gamma^2 E_0 \ln(r/a)$
$(+\frac{1}{2}, \pm\frac{1}{2})$ and $(-\frac{1}{2}, \pm\frac{1}{2})$	$+(1/8)\gamma^2 E_0 \ln(r/a)$

Coupling of half-vortex cores



For attraction of polaritons with opposite spin, $\alpha_2 = U_0 - 2U_1 < 0$, one has the attraction of half-vortex cores.

Coupling of half-vortex cores



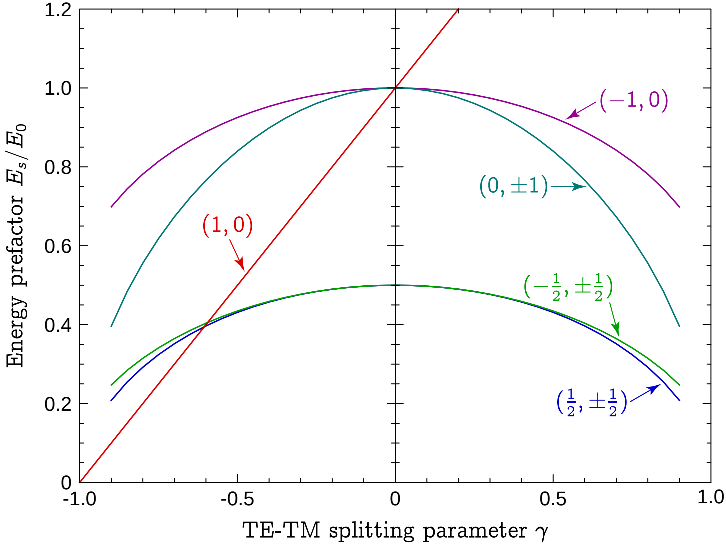
$(0, \pm 1) \rightarrow (+\frac{1}{2}, \pm\frac{1}{2})$ and $(-\frac{1}{2}, \pm\frac{1}{2})$, stable.

$(-1, 0) \rightarrow (-\frac{1}{2}, +\frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$, metastable.

$(+1, 0) \rightarrow (+\frac{1}{2}, +\frac{1}{2})$ and $(+\frac{1}{2}, -\frac{1}{2})$, $\gamma > 0$: unstable, $\gamma < 0$: stable.

Energies of warped vortices

Vortex energy $E_{\text{vor}} = E_c + E_s \ln(R/a)$, and $E_0 = \pi\hbar^2 n/m^*$.



BKT transition temperature

The energy of a vortex $E_{\rm vor} = E_c + E_s \ln(R/a)$.
The free energy [J. M. Kosterlitz and D. J. Thouless (1973); J. M. Kosterlitz (1974)]

$$F = E_s \ln(R/a) - TS = E_s \ln(R/a) - T \ln(R^2/a^2) = (E_s - 2T) \ln(R/a),$$

so that single vortices appear and destroy the order at $T_c = \frac{1}{2} E_s$.

Four half-vortices:

$$F = 2(E_s^{\rm star} + E_s^{\rm lemon}) \ln(R/a) - 4T \ln(R^2/a^2), \qquad T_c = \frac{1}{4}(E_s^{\rm star} + E_s^{\rm lemon}).$$

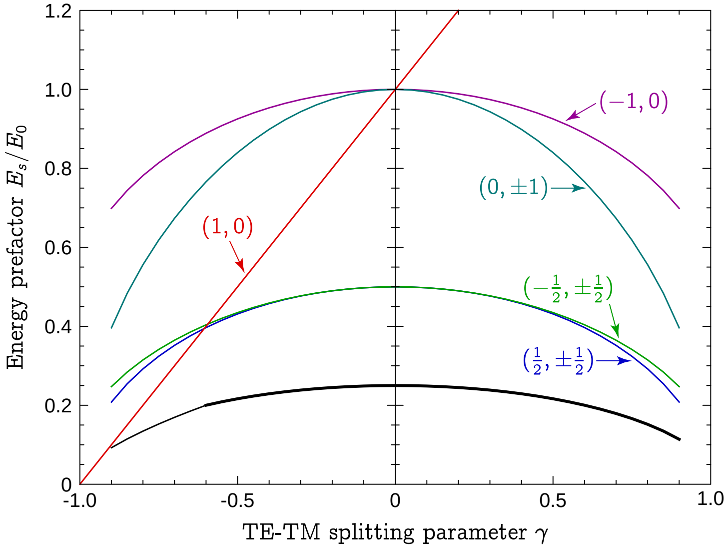
Vortex molecules $(-\frac{1}{2}, \frac{1}{2}) - (1,0) - (-\frac{1}{2}, -\frac{1}{2})$:

$$F = (2E_s^{\rm star} + E_s^{(1,0)}) \ln(R/a) - 3T \ln(R^2/a^2), \qquad T_c = \frac{1}{6}(2E_s^{\rm star} + E_s^{(1,0)}).$$

Crossover at $E_s^{(1,0)} = \frac{1}{2} \left(3E_s^{\rm lemon} - E_s^{\rm star}\right)$.

BKT transition temperature

Vortex energy $E_{\text{vor}} = E_c + E_s \ln(R/a)$, and $E_0 = \pi\hbar^2 n/m^*$.

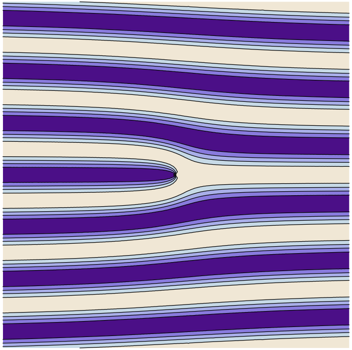


For $m_l \gg m_t$, i.e, for $\gamma \to -1$, the phase transition is defined by proliferation of vortex molecules $(-\frac{1}{2},\frac{1}{2}) - (1,0) - (-\frac{1}{2},-\frac{1}{2})$.

Observation of vortices

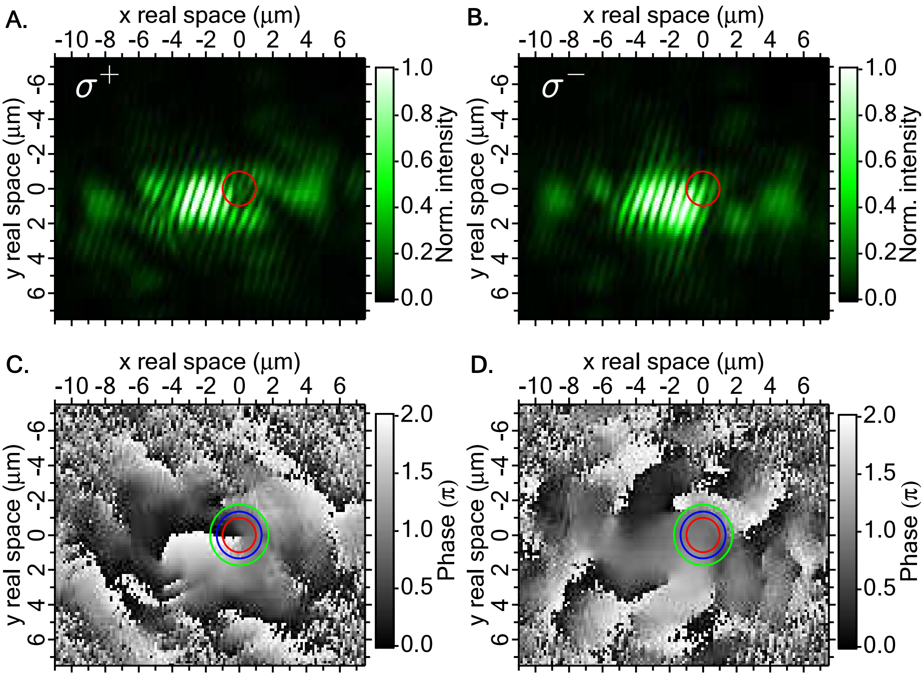
For one-component condensate one observes the interference pattern of two beams emitted by the same condensate.
One with vortex and the other without but inclined (plane wave):

$$|f(r)e^{i\phi} + e^{i\kappa y}|^2$$



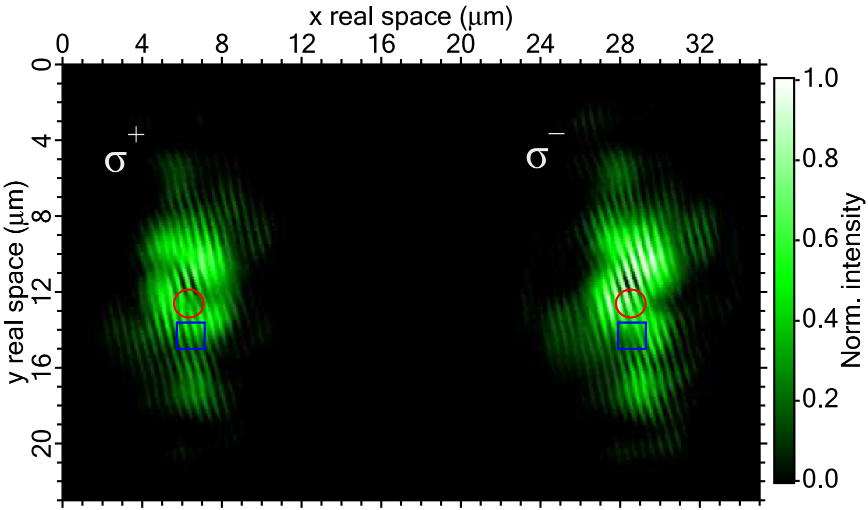
For polarized condensate one studies the interference patterns in both circular polarizations.
HQV: fork in one circular polarization and regular fringes in the other.

Observation of half-vortices



K. G. Lagoudakis *et al.*, *Science* 326, 974 (2009).

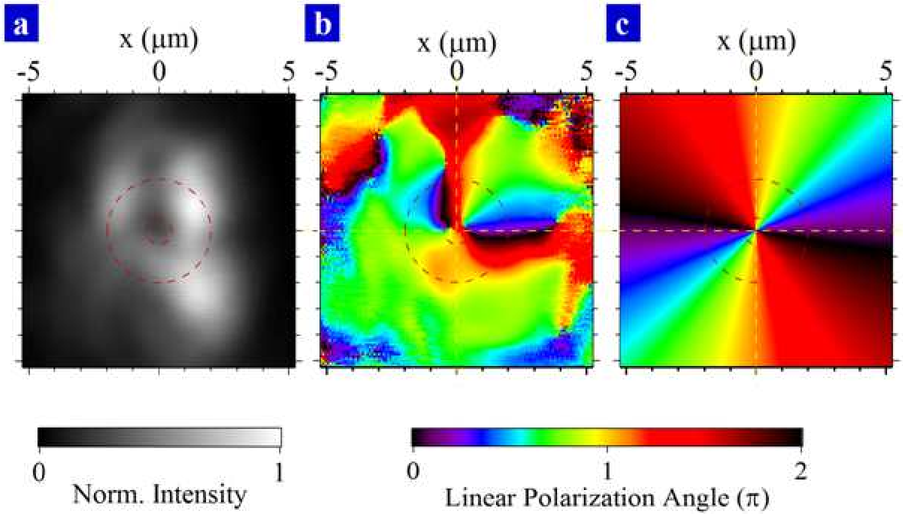
Observation of half-vortices



Close pair of $(-1/2, +1/2)$ (in red circle) and $(+1/2, +1/2)$ (in blue box). These HQV form pure phase vortex $(0, +1)$ when placed together. Their close position is an indication of weak polarization pinning.

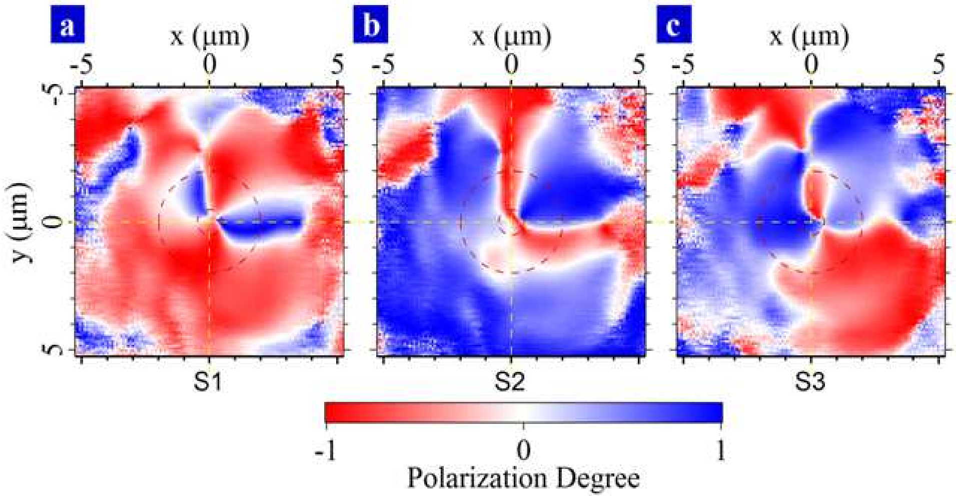
K. G. Lagoudakis *et al.*, *Science* 326, 974 (2009).

Observation of hyperbolic spin vortex (-1,0)



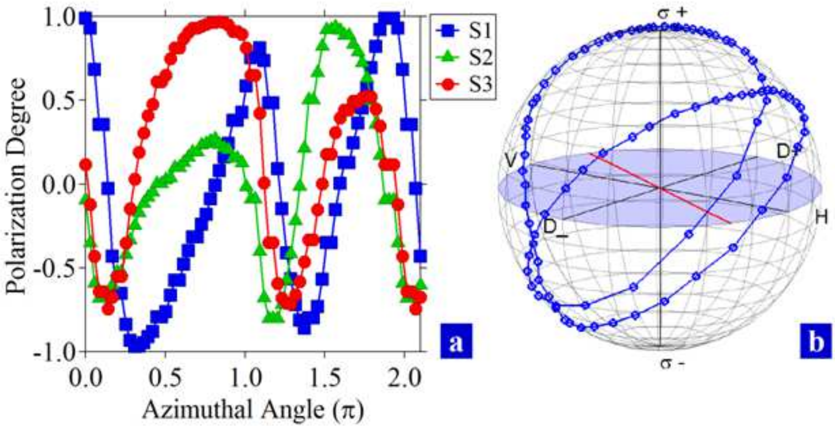
F. Manni, Y. Léger, Y. G. Rubo, R. André, B. Deveaud, *Nature Commun.* (2013).

Observation of hyperbolic spin vortex (-1,0)



F. Manni, Y. Léger, Y. G. Rubo, R. André, B. Deveaud, *Nature Commun.* (2013).

Observation of hyperbolic spin vortex (-1,0)



The spin-vortex $(-1, 0)$ is metastable: $(-1, 0) \to (-\frac{1}{2}, \frac{1}{2}) + (-\frac{1}{2}, -\frac{1}{2})$.

F. Manni, Y. Léger, Y. G. Rubo, R. André, B. Deveaud, *Nature Commun.* (2013).

Acknowledgments and References

Collaboration:

- A. Kavokin, T. Ostatnický (Southampton, UK)
- Tim Liew (Singapur)
- M. Toledo-Solano (México)
- F. Manni, Y. Léger, K. Lagoudakis, B. Deveaud-Plédran (Lausanne, Swiss)

- Y.G. Rubo, *Phys. Rev. Lett.* 99, 106401 (2007).
- T.C.H. Liew, Y.G. Rubo, A.V. Kavokin, *Phys. Rev. Lett.* 101, 187401 (2008).
- K.G. Lagoudakis, T. Ostatnický, A.V. Kavokin, Y.G. Rubo, R. André, B. Deveaud-Plédran, *Science* 326(5955), 974 (2009).
- M. Toledo Solano, Y.G. Rubo, *J. Phys. Conf. Ser.* 210, 012024 (2010); *Phys. Rev. B* 82, 127301 (2010).
- Y.G. Rubo, in *Physics of Quantum Fluids*, Springer Series in Solid-State Sciences 177, Springer (2013).
- F. Manni, Y. Léger, Y. G. Rubo, R. André, B. Deveaud, *Nature Commun.* (2013).

Contents