Light-matter coupling: from the weak to the ultrastrong coupling and beyond

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QUANTUM LIGHT & MATTER

General theory

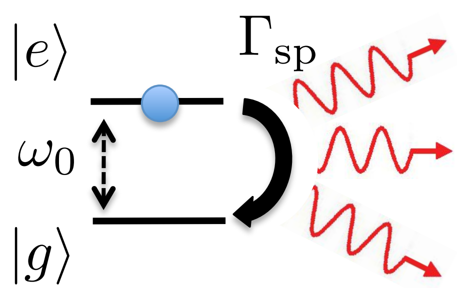
Open quantum systems

Material implementations

Advanced topics

General theory

Light-matter coupling: free space



Emission rate with Fermi golden rule:

$$\Gamma_{\mathrm{sp}} = \frac{\omega_0^3 d_{ge}^2}{3\pi\epsilon_0 \hbar c^3}$$

We can derive such a formula from the minimal coupling Hamiltonian:

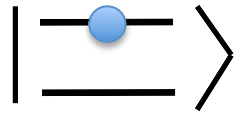
$$H = H_{\mathrm{field}} + \frac{(\mathbf{p} - e\mathbf{A}(\mathbf{r}))^2}{2m} + V(\mathbf{r})$$

$$H = \underbrace{H_{\mathrm{field}} + \frac{\mathbf{p}^2}{2m} + V(\mathbf{r})}_{H_0} \underbrace{- \frac{e\mathbf{p}\mathbf{A}(\mathbf{r})}{m} + \frac{e^2\mathbf{A}(\mathbf{r})^2}{2m}}_{H_{\mathrm{int}}}$$

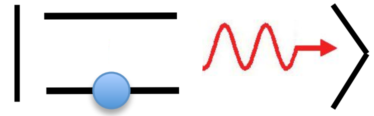
Light-matter coupling: free space

Fermi golden rule: $\Gamma_{\rm sp} = \dfrac{2\pi}{\hbar}|\langle i|H_{\rm int}|f\rangle|^2\rho(\hbar\omega_0) = \dfrac{\omega_0^3 d_{ge}^2}{3\pi\epsilon_0\hbar c^3}$

Initial state: $|i\rangle = |e\rangle \otimes |0\rangle$ =



Final state: $|f\rangle = |g\rangle \otimes |1\rangle$ =



Interaction Hamiltonian:

$$H_{\rm int} \quad = \quad \Omega_R(a^\dagger + a)(|e\rangle\langle g| + |g\rangle\langle e|) + \frac{\Omega_R^2}{\omega_0}(a^\dagger + a)(a^\dagger + a)$$

With $\Omega_R = \sqrt{\dfrac{\hbar\omega_0 d_{\rm ge}^2}{2\epsilon_0 V}}$ "Vacuum Rabi frequency"

Density of photonic states: $\rho(\hbar\omega_0) = \dfrac{V\omega_0^2}{3\pi^2\hbar c^3}$

Rotating wave approximation

Antiresonant terms

Connect states whose energy difference is $\simeq 2\omega_0$

Do not contribute in the Fermi golden rule

$$H_{\rm int} = \Omega_R(a^\dagger + a)(|e\rangle\langle g| + |g\rangle\langle e|) + \frac{\Omega_R^2}{\omega_0}(a^\dagger + a)(a^\dagger + a)$$

Resonant terms

Connect states whose energy difference is $\simeq 0$

Do contribute in the Fermi golden rule

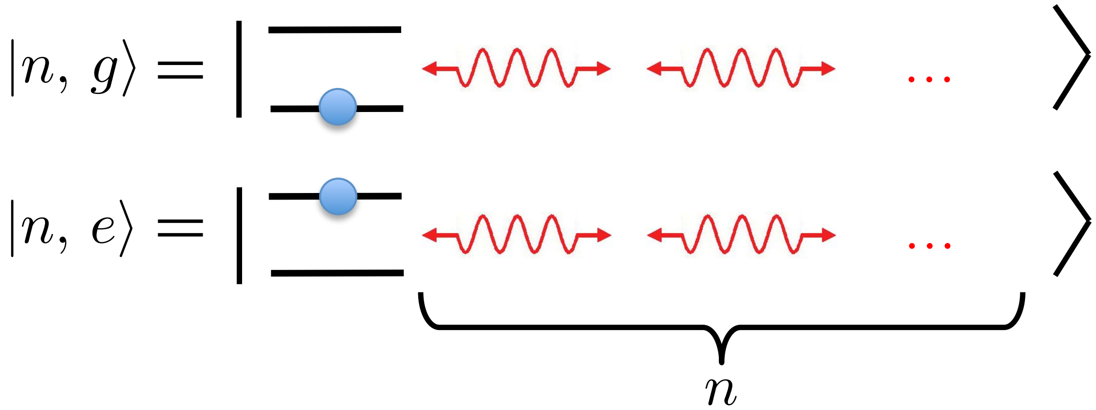
We can thus consider the simpler Hamiltonian:

$$H_{\rm int}^{\rm RWA} = \Omega_R(a^\dagger|g\rangle\langle e| + a|e\rangle\langle g|) + \frac{2\Omega_R^2}{\omega_0}a^\dagger a$$

Emission Absorption Photon renormalisation

Jaynes-Cummings model

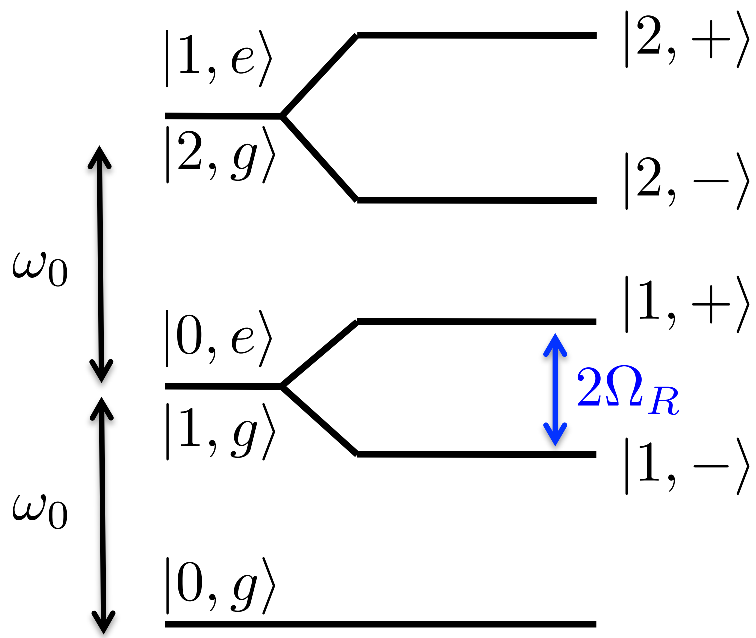
$$H_{\mathrm{JC}} = \omega_c a^\dagger a + \omega_0 |e\rangle\langle e| + \Omega_R (a^\dagger |g\rangle\langle e| + a|e\rangle\langle g|)$$



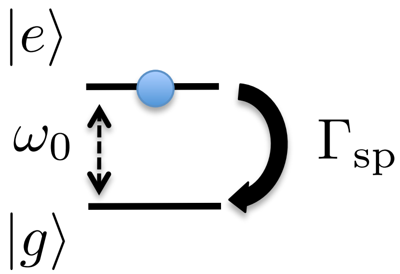
$|n,\, g\rangle$ and $|n-1,\, e\rangle$ form a closed subspace:

$$H^n_{\mathrm{JC}} = \begin{bmatrix} \omega_c & \sqrt{n}\Omega_R \\ \sqrt{n}\Omega_R & \omega_0 \end{bmatrix}$$

whose eigenvalues are $|n,\, -\rangle$ and $|n,\, +\rangle$, split at resonance of $2\sqrt{n}\Omega_R$

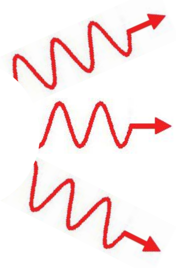


Purcell effect

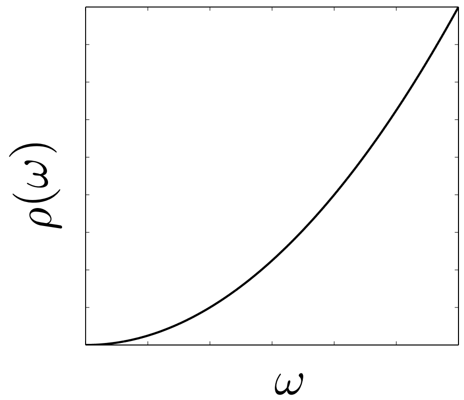


$$\Gamma_{\mathrm{sp}} = \frac{2\pi}{\hbar} |\langle i | H_{\mathrm{int}} | f \rangle|^2 \rho(\hbar\omega_0)$$

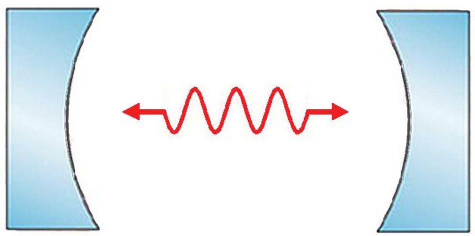
Photonic density of states



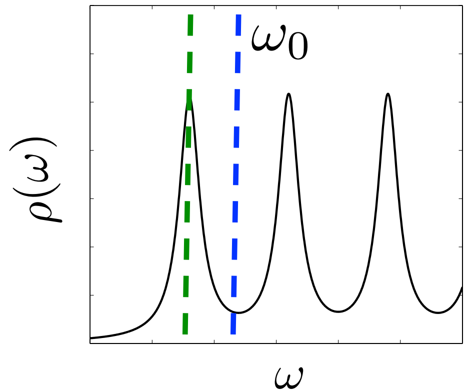
Free space:



$\Gamma_{\mathrm{sp}} \propto \omega_0^3$



Cavity:



Enhancement

Suppression

Purcell effect

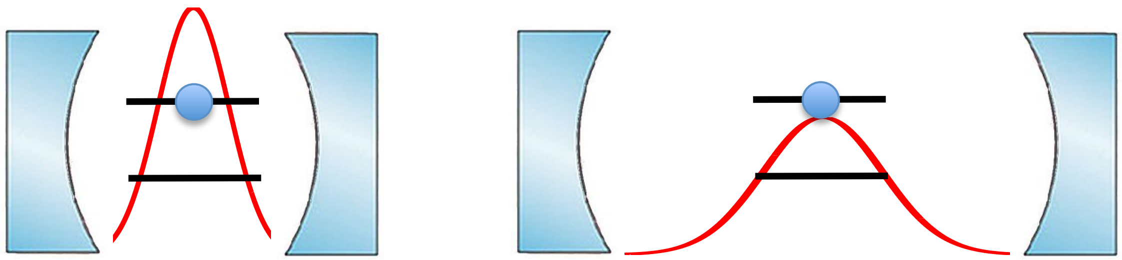
The enhancement is given by the ratio between the densities of states in free space and inside the cavity

Quality factor of the cavity

$$F_{\mathrm{P}} = \frac{3}{4\pi^2}\left(\frac{\lambda}{n}\right)^3\left(\frac{Q}{V_{\mathrm{eff}}}\right)$$

Volume of the cavity

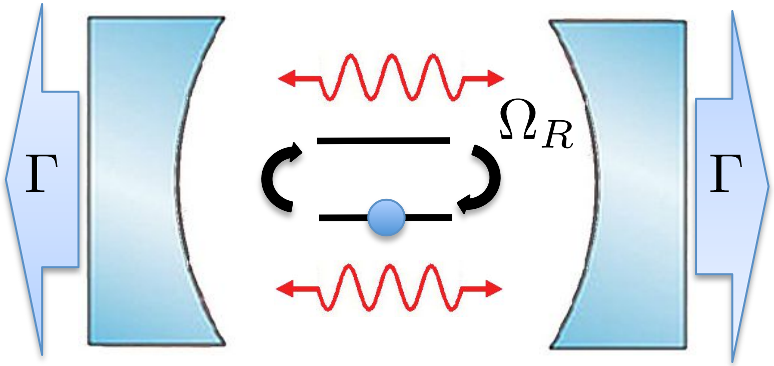
Mode confinement: smaller cavity = larger coupling



$$\Omega_R \propto \frac{1}{\sqrt{V_{\mathrm{eff}}}}$$

Importance of sub-wavelength confinement!

Toward strong coupling

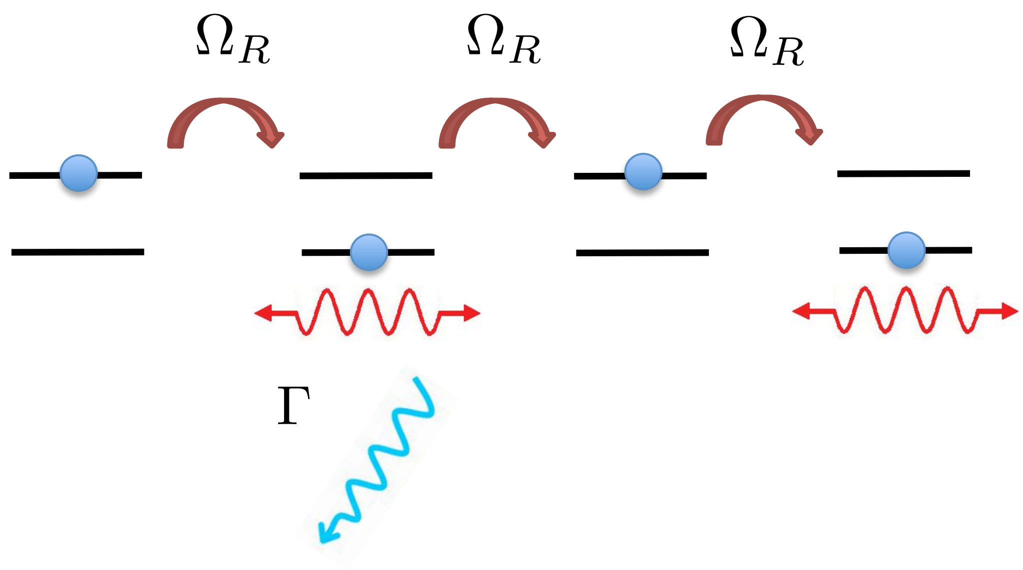


Fermi golden rule: first order perturbation.

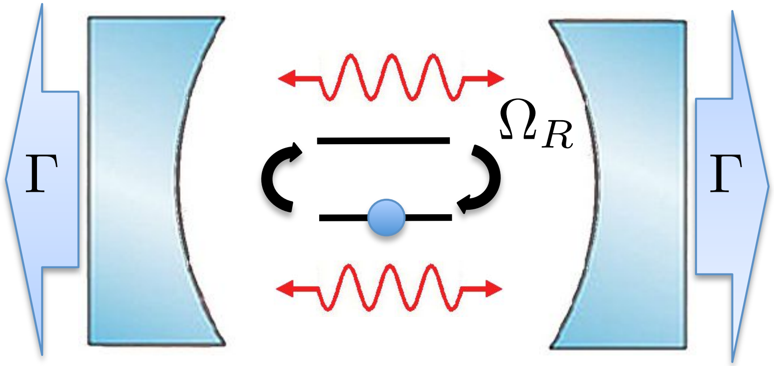
It cannot account for higher order processes, *i.e.* reabsorption.

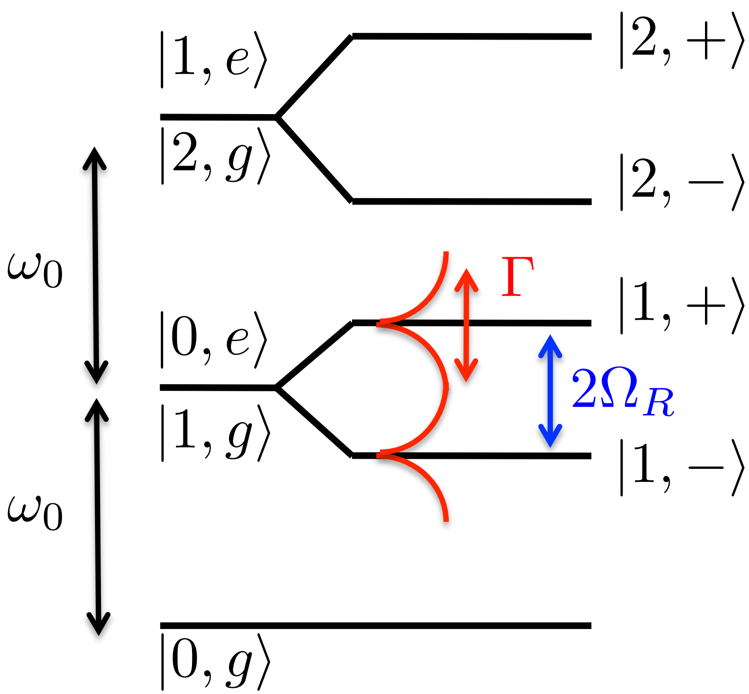
Valid if $\Omega_R < \Gamma$

If $\Omega_R > \Gamma$ the emitted photons is trapped long enough to be reabsorbed



Strong coupling





The coupling splits the degenerate levels, creating the Jaynes-Cummings ladder.

The losses give the resonances a finite width

Strong coupling: $\Omega_R > \Gamma$

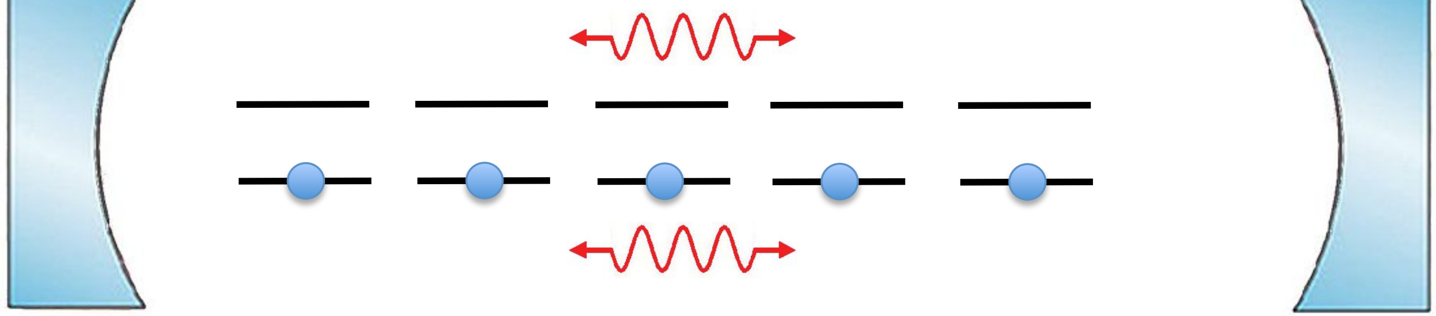
Condition to spectroscopically resolve the resonant splitting.

In the strong coupling regime we cannot consider transitions between uncoupled modes, *e.g.*, $|0, e\rangle \longrightarrow |1, g\rangle$.

We are obliged to consider the dressed states, $|1, -\rangle$, $|1, +\rangle$, etc…

The Dicke model

$N \gg 1$ two level systems in a cavity



$$H_{\text{Dicke}} = \omega_c a^\dagger a + \sum_{j=1}^{N} \omega_0 |e_j\rangle\langle e_j| + \Omega_R (a^\dagger |g_j\rangle\langle e_j| + a|e_j\rangle\langle g_j|)$$

All the two level systems couple to the same photonic field
We can introduce coherent excitation operators

$$b = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |g_j\rangle\langle e_j|$$

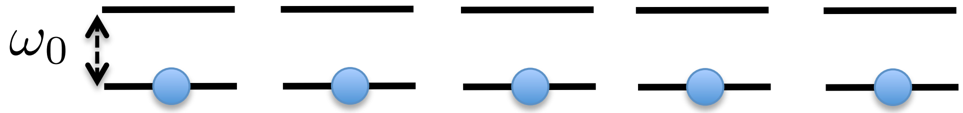
State with n systems in the excited state

$$\langle n|[b,\, b^\dagger]|n\rangle = 1 - \frac{2n}{N}$$

Bosons in the limit $N \gg n$

Bosons for real

$N \gg 1$ *distinguishable* two level systems



Partition function: $$Z = (1 + e^{-\beta\omega_0})^N = \sum_{m=0}^{N} \binom{N}{m} e^{-m\beta\omega_0}$$

If they are *indistinguishable* instead:

Partition function of a bosonic field

$$Z = \sum_{m=0}^{N} \cancel{\binom{N}{m}} e^{-m\beta\omega_0} = \sum_{m=0}^{N} e^{-m\beta\omega_0} \to \frac{1}{1 - e^{-\beta\omega_0}}$$

"With *n* photons we cannot distinguish a *n* level system from a bosonic one"

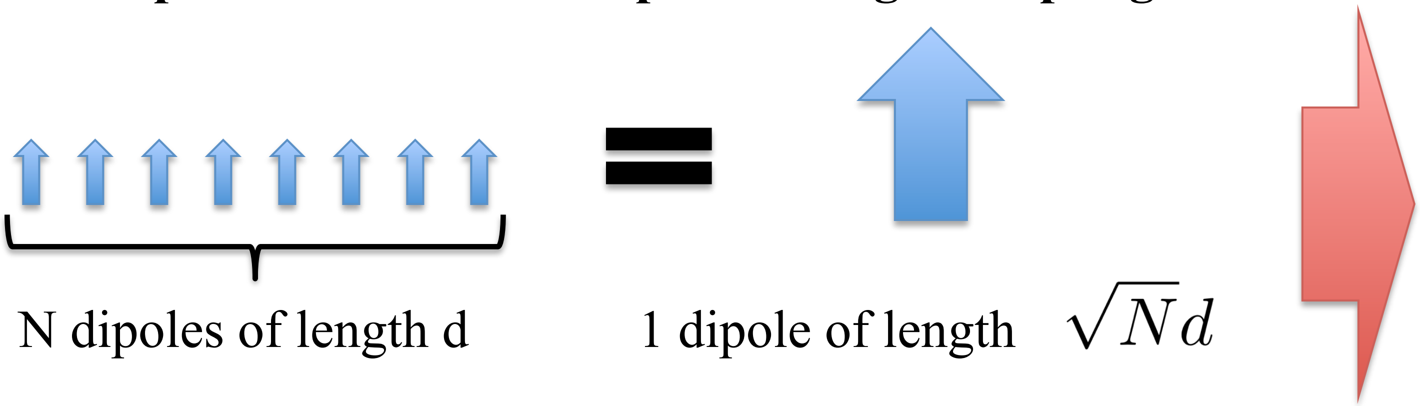
The Dicke model

From the Dicke model: $$H_{\text{int}} = \sum_{j=1}^{N} \Omega_R (a^\dagger |g_j\rangle\langle e_j| + a|e_j\rangle\langle g_j|)$$

We can then substitute the coherent operators: $$b = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |g_j\rangle\langle e_j|$$

Obtaining: $$H_{\text{int}} = \sqrt{N}\Omega_R (a^\dagger b + b^\dagger a)$$

Superradiance: more dipoles = larger coupling



$$\Omega_R \propto \sqrt{N}$$

R. H. Dicke, Phys. Rev. ***93****, 99 (1954)*

The Polariton

The full light-matter Hamiltonian has the form:

$$H_{\text{Dicke}} = \omega_c a^\dagger a + \omega_0 b^\dagger b + \Omega_R (a^\dagger b + b^\dagger a)$$

Introducing the polaritonic operators: $p_j = x_j a + y_j b, \quad j \in [\text{LP,UP}]$

With $|x_j|^2 + |y_j|^2 = 1$, in order to have $[p_j \, , p_i^\dagger] = \delta_{i,j}$

We can diagonalise the Hamiltonian as: $H = \sum_{j \in [\text{LP,UP}]} \omega_j p_j^\dagger p_j$

A linear system of N interacting bosonic fields is (almost) always equivalent to a system of N non-interacting bosonic fields

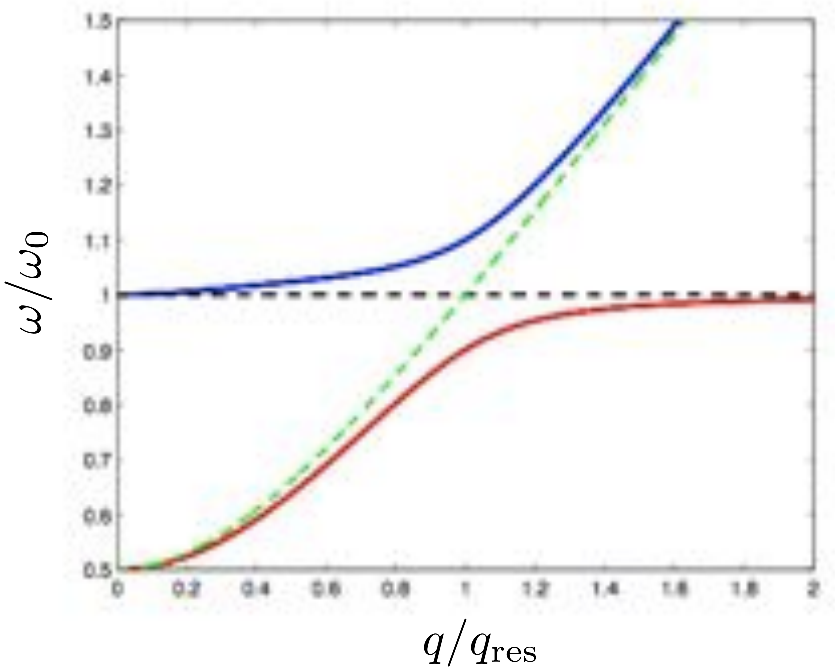
J. J. Hopfield, Phys. Rev. ***112****, 1555 (1958)*

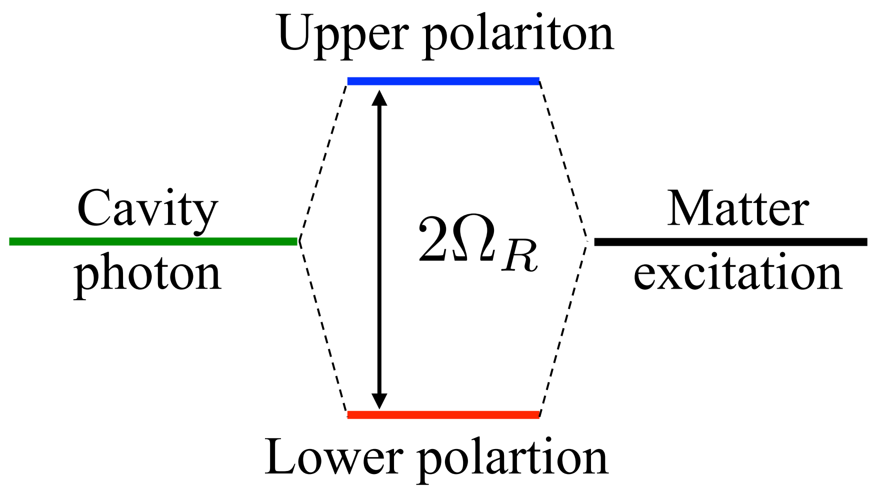
The Polariton

$$p^\dagger|0\rangle = x|\;\rangle + y|\;\rangle$$

Half light and half matter excitation

Modes that are:
- easy to excite and observe
- interact strongly





Ultrastrong coupling

$\Omega_R > \Gamma$ We can have either

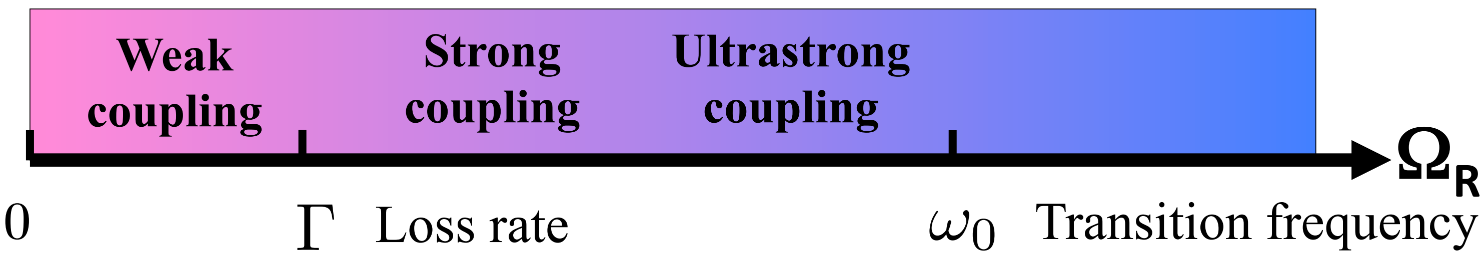
- Large Ω_R (strong coupling)
- Small Γ (good cavity)

$\dfrac{\Omega_R}{\omega_0}$ Is the relevant small parameter in perturbation theory
It measures the "intrinsic" strength of the transition

When $\dfrac{\Omega_R}{\omega_0} \simeq 1$ we can expect non-perturbative effects

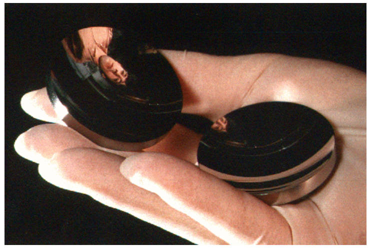
Ultrastrong coupling regime ($\dfrac{\Omega_R}{\omega_0} > 0.1$)

Fermi Golden rule Polaritons New physics



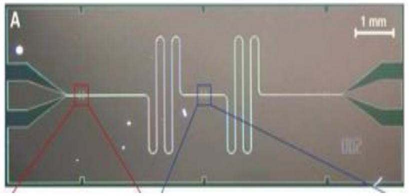
Comparison with other systems

Atoms in superconducting cavities



$$\frac{\Omega_R}{\omega_0} < 0.000001$$

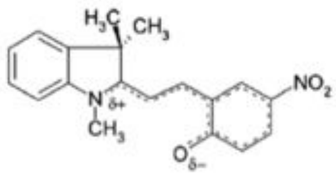
Superconducting circuits (2010)



$$\frac{\Omega_R}{\omega_0} = 0.12$$

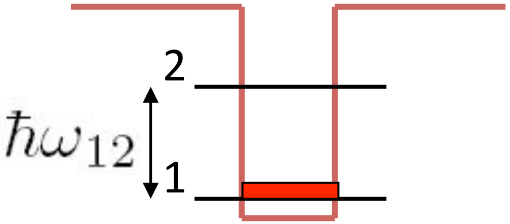
Organic molecules (2011)

Merocyanine



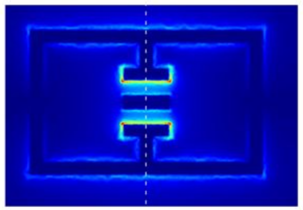
$$\frac{\Omega_R}{\omega_0} = 0.16$$

Intersubband polaritons (2009)



$$\frac{\Omega_R}{\omega_0} = 0.24$$

Landau polaritons (2012)



$$\frac{\Omega_R}{\omega_0} = 0.58$$

Higher density superradiance
Intrinsically larger dipoles
Better confinement

Ultrastrong coupling

Let us consider the bosonised light-matter Hamiltonian
without the rotating wave approximation:

$$H = \underbrace{\omega_c a^\dagger a + \omega_0 b^\dagger b}_{H_0} + \underbrace{\Omega_R (a^\dagger + a)(b^\dagger + b) + \frac{\Omega_R^2}{\omega_0}(a^\dagger + a)^2}_{H_{\text{int}}}$$

First order perturbation: $\Delta E_\phi^{(1)} \propto \Omega_R$

The second order contribution is due to antiresonant terms ($ab,\ a^\dagger b^\dagger$, etc…)

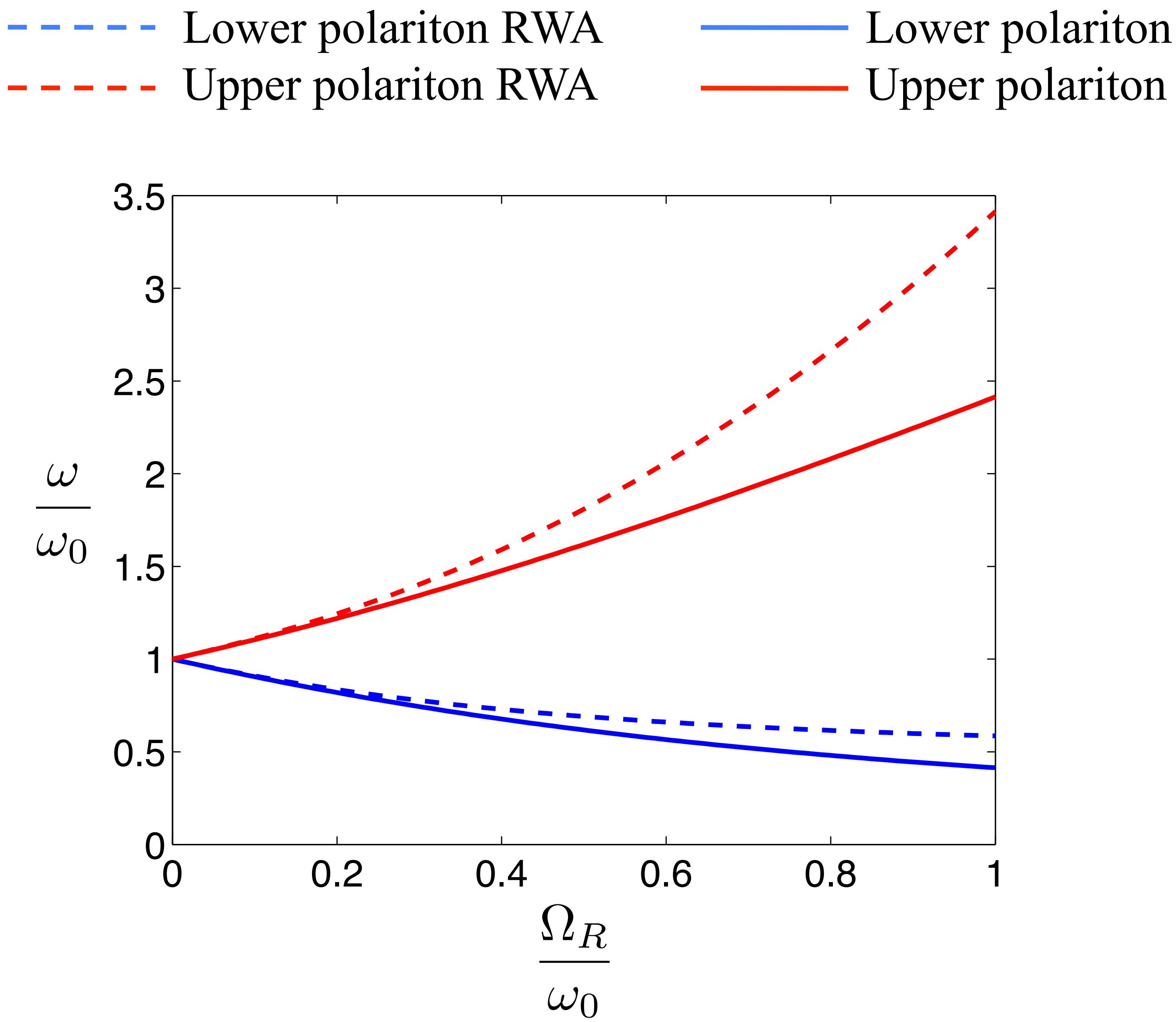
$\propto \Omega_{\mathrm{R}}^2$

Second order perturbation: $$\Delta E_\phi^{(2)} = \sum_{|\psi\rangle \neq |\phi\rangle} \frac{|\langle \phi | H_{\text{int}} | \psi \rangle|^2}{E_\phi - E_\psi} \propto \frac{\Omega_R^2}{\omega_0}$$

$\propto \omega_0$

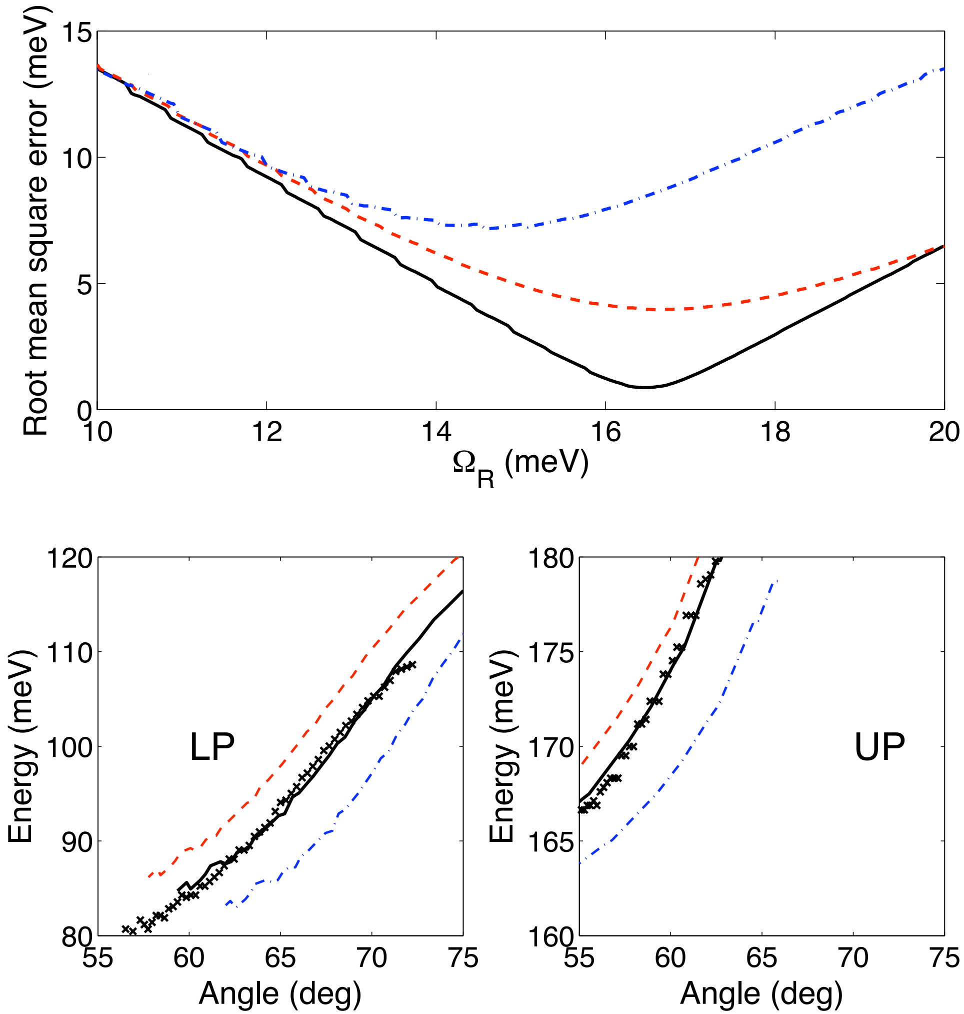
In the ultrastrong coupling regime the antiresonant terms are not negligible!

Ultrastrong coupling



Ultrastrong coupling

Data fitted with different Hamiltonians, with Ω_R as only free parameter



$$H_{\text{int}} = \Omega_R(a^\dagger + a)(b^\dagger + b) + \frac{\Omega_R^2}{\omega_0}(a^\dagger + a)^2$$

$$H_{\text{int}}^{\text{RWA}} = \Omega_R(a^\dagger b + b^\dagger a) + \frac{2\Omega_R^2}{\omega_0} a^\dagger a$$

$$H_{\text{int}}^{\text{RWA'}} = \Omega_R(a^\dagger b + b^\dagger a)$$

The best fit gives: $\dfrac{\Omega_R}{\omega_0} = 0.11$

First observation of the ultrastrong coupling regime

*A. Anappara et al., Phys. Rev. B **79**, 201303(R) (2009)*

Ultrastrong coupling

We have thus to consider the full light-matter Hamiltonian:

$$H = \omega_c a^\dagger a + \omega_0 b^\dagger b + \Omega_R (a^\dagger + a)(b^\dagger + b) + \frac{\Omega_R^2}{\omega_0}(a^\dagger + a)^2$$

Can we put it in diagonal form?

$$H = \sum_{j \in [\mathrm{LP},\mathrm{UP}]} \omega_j p_j^\dagger p_j$$

The previous transformation: $p_j = x_j a + y_j b$ is not enough, as we cannot generate the antiresonant terms multiplying $p_j^\dagger$ and p_j

We need instead a transformation that mixes creation and annihilation operators:

$$p_j = x_j a + y_j b + z_j a^\dagger + w_j b^\dagger$$

In order to have $[p_j \, , p_i^\dagger] = \delta_{i,j}$, the coefficients have to respect the normalisation condition $|x_j|^2 + |y_j|^2 - |z_j|^2 - |w_j|^2 = 1$

The minuses imply that the coefficients **are not bounded!**

Ultrastrong coupling

The ground state is the state annihilated by the annihilation operators

We call $|0\rangle$ the ground state of the uncoupled light-matter system

$$a|0\rangle = b|0\rangle = 0$$

From the decomposition $p_j = x_j a + y_j b + z_j a^\dagger + w_j b^\dagger$

$$p_j|0\rangle \neq 0$$

The coupling modifies the ground state

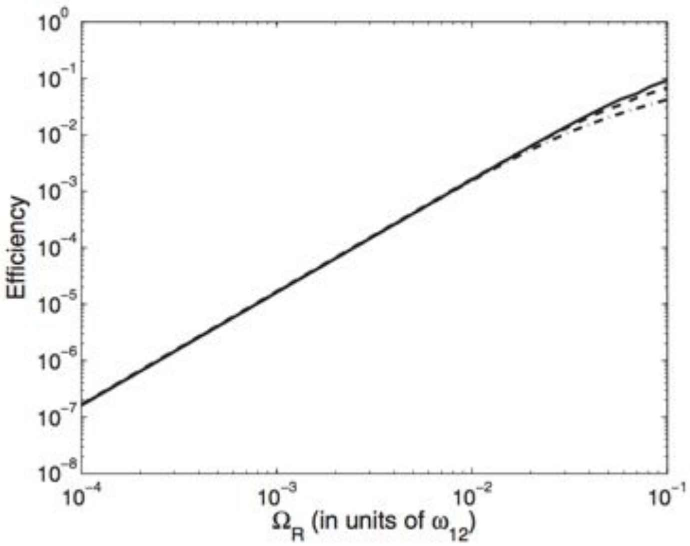
We introduce the ground state of the coupled system $|G\rangle$

$$p_j|G\rangle = 0$$

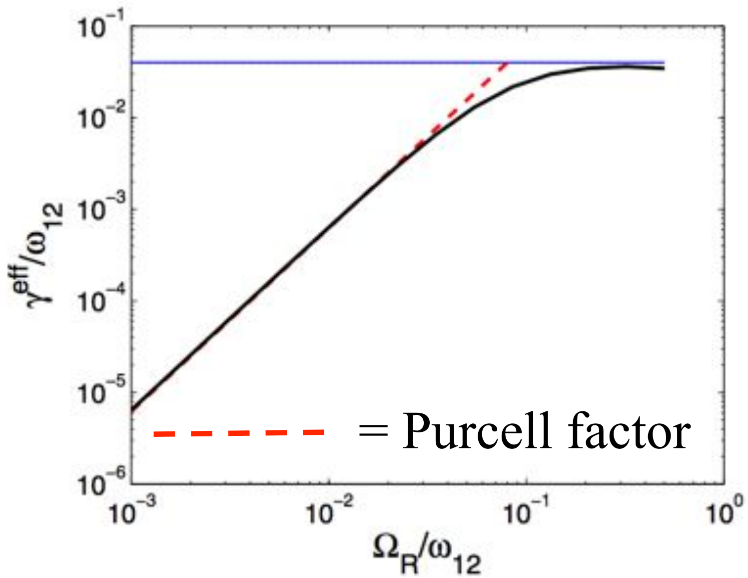
We have then $\langle G|a^\dagger a|G\rangle = |z_{\mathrm{LP}}|^2 + |z_{\mathrm{UP}}|^2 \neq 0 \propto \frac{\Omega_R^2}{\omega_0^2} + O(\frac{\Omega_R^3}{\omega_0^3})$

The ground state contains a population of bound photons

What about the Purcell effect?



S. De Liberato and C. Ciuti, Phys. Rev. B 77, 155321 (2008)



*C. Ciuti and I. Carusotto, Phys. Rev. A **74**, 033811 (2006)*

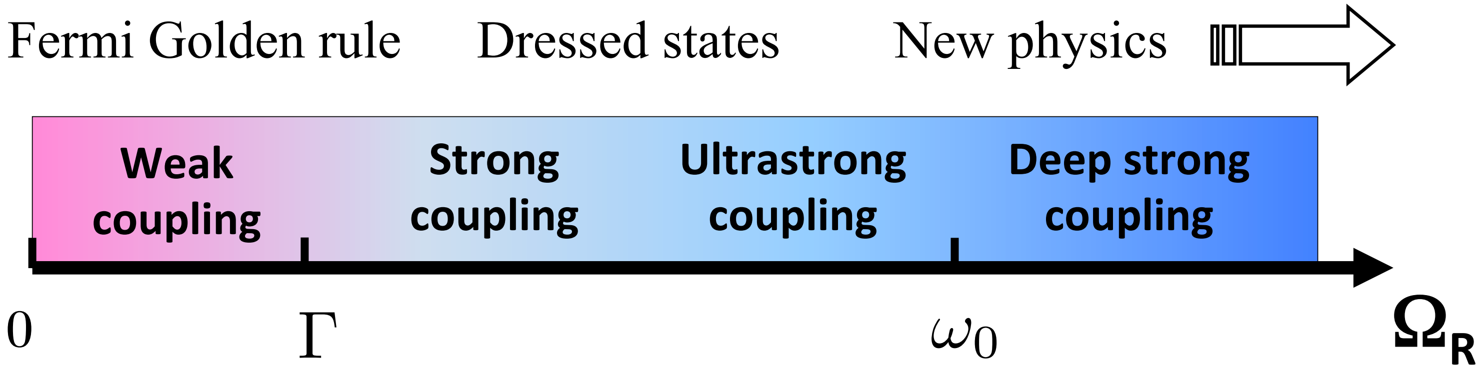
Strong coupling regime: quadratic dependency upon Ω_R

Ultrastrong coupling regime: saturation

…and beyond

What happens if $\dfrac{\Omega_R}{\omega_0} > 1$?

The coupling becomes completely non-perturbative
It has been called deep strong coupling regime



Do we expect qualitatively new phenomena?

J. Casanova, et al., Phys. Rev. Lett. ***105****, 263603 (2010)*

…and beyond

If $\dfrac{\Omega_R}{\omega_0} > 1$ the last term, **always positive**, becomes dominant

$$H = \omega_c a^\dagger a + \omega_0 b^\dagger b + \Omega(a^\dagger + a)(b^\dagger + b) + \frac{\Omega^2}{\omega_0}(a^\dagger + a)^2$$

$$H = H_{\text{field}} + \frac{\mathbf{p}^2}{2m} + V(\mathbf{r}) - \frac{e\mathbf{p}\mathbf{A}(\mathbf{r})}{m} + \frac{e^2\mathbf{A}(\mathbf{r})^2}{2m}$$

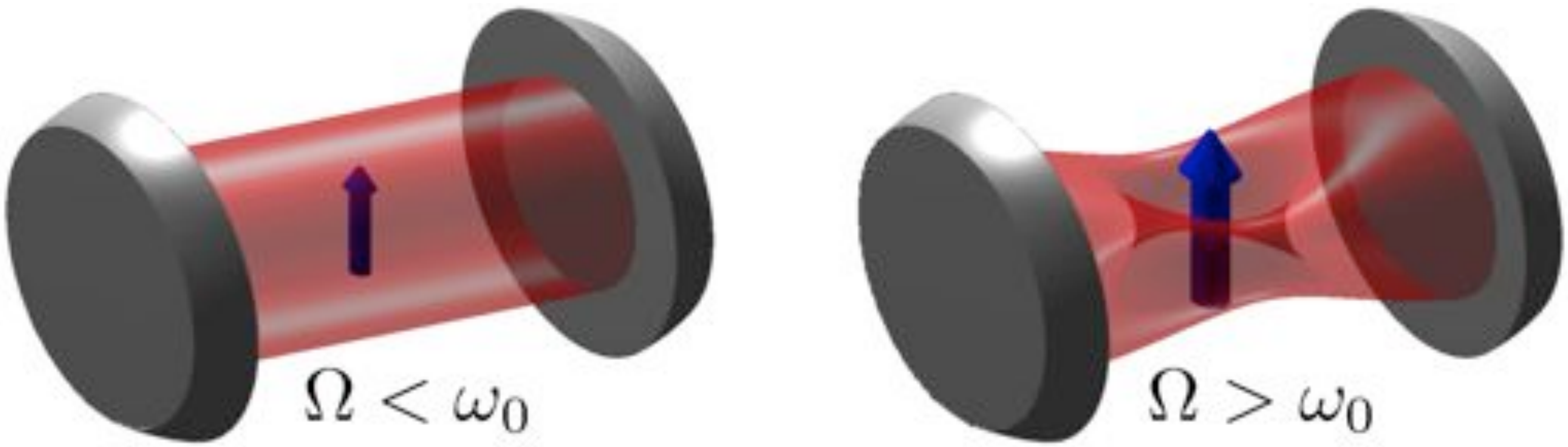
Intensity of the field at the location of the dipoles

The low energy modes need to minimize the field location over the dipoles

Polariton modes will be
- pure photon modes that avoid the dipoles
- pure matter mode

Light and matter decouple in the deep strong coupling regime

…and beyond

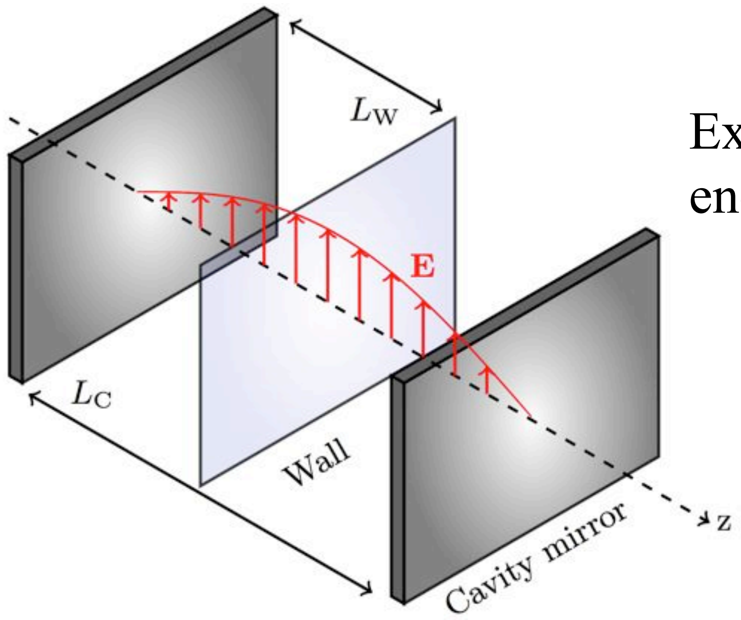


The photonic field avoids the dipoles

Light-matter interaction is due to **local** interactions

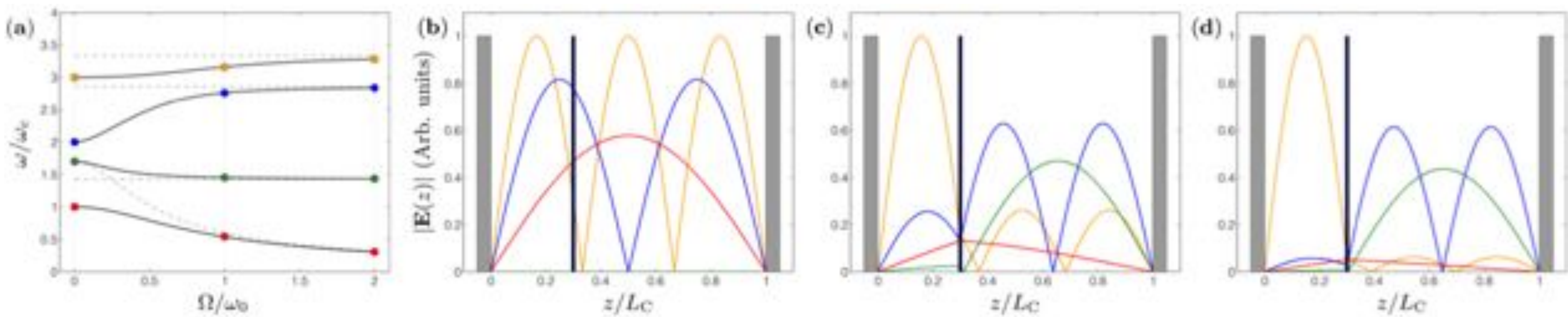
Light and matter do not exchange energy

An example

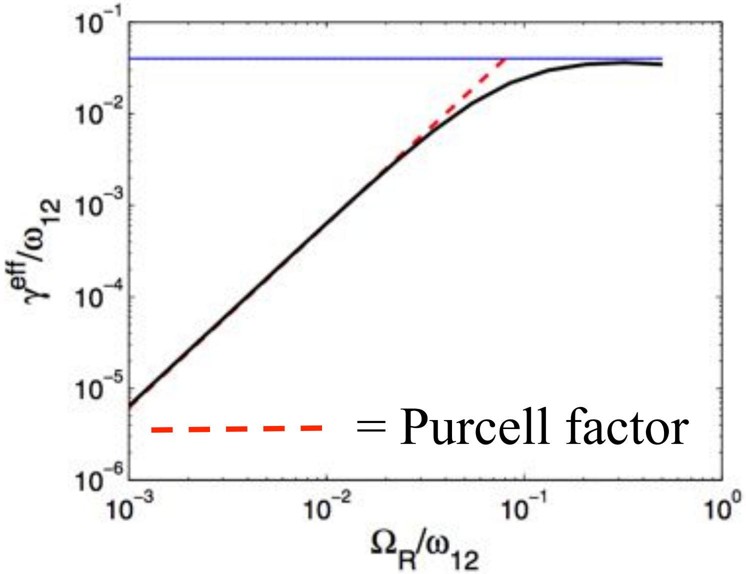


Example: a two-dimansional metallic cavity enclosing a wall of in-plane dipoles

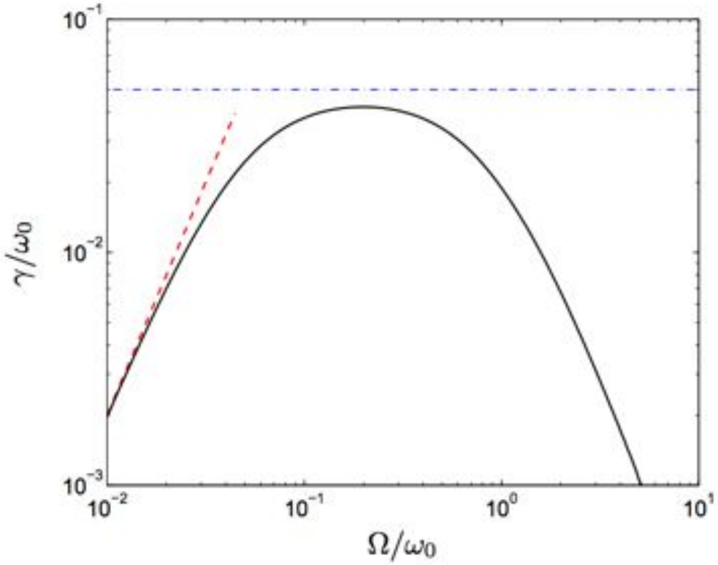
The wall becomes a metallic mirror



What about the Purcell effect?



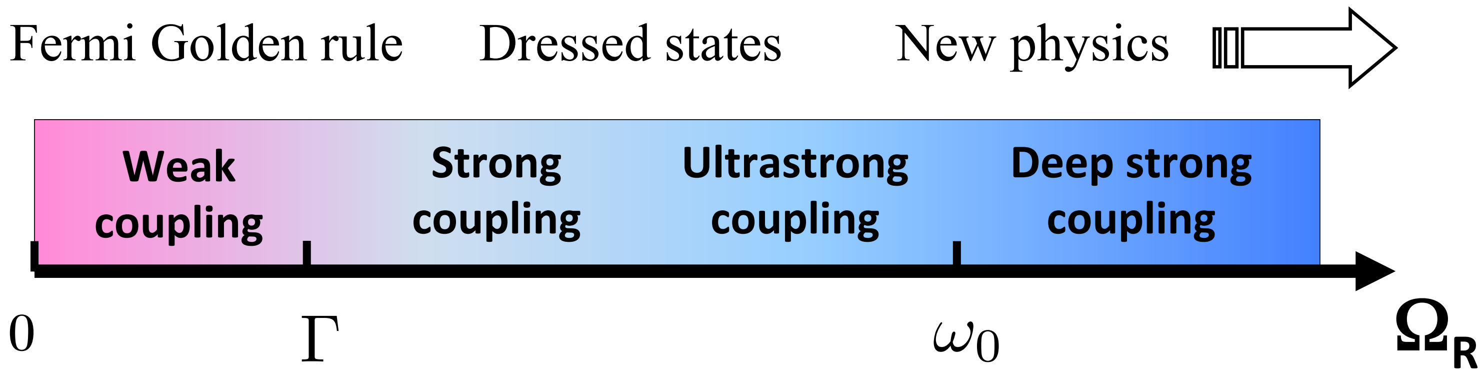
C. Ciuti and I. Carusotto,
Phys. Rev. A ***74****, 033811 (2006)*



S. De Liberato,
arXiv:1308.2812

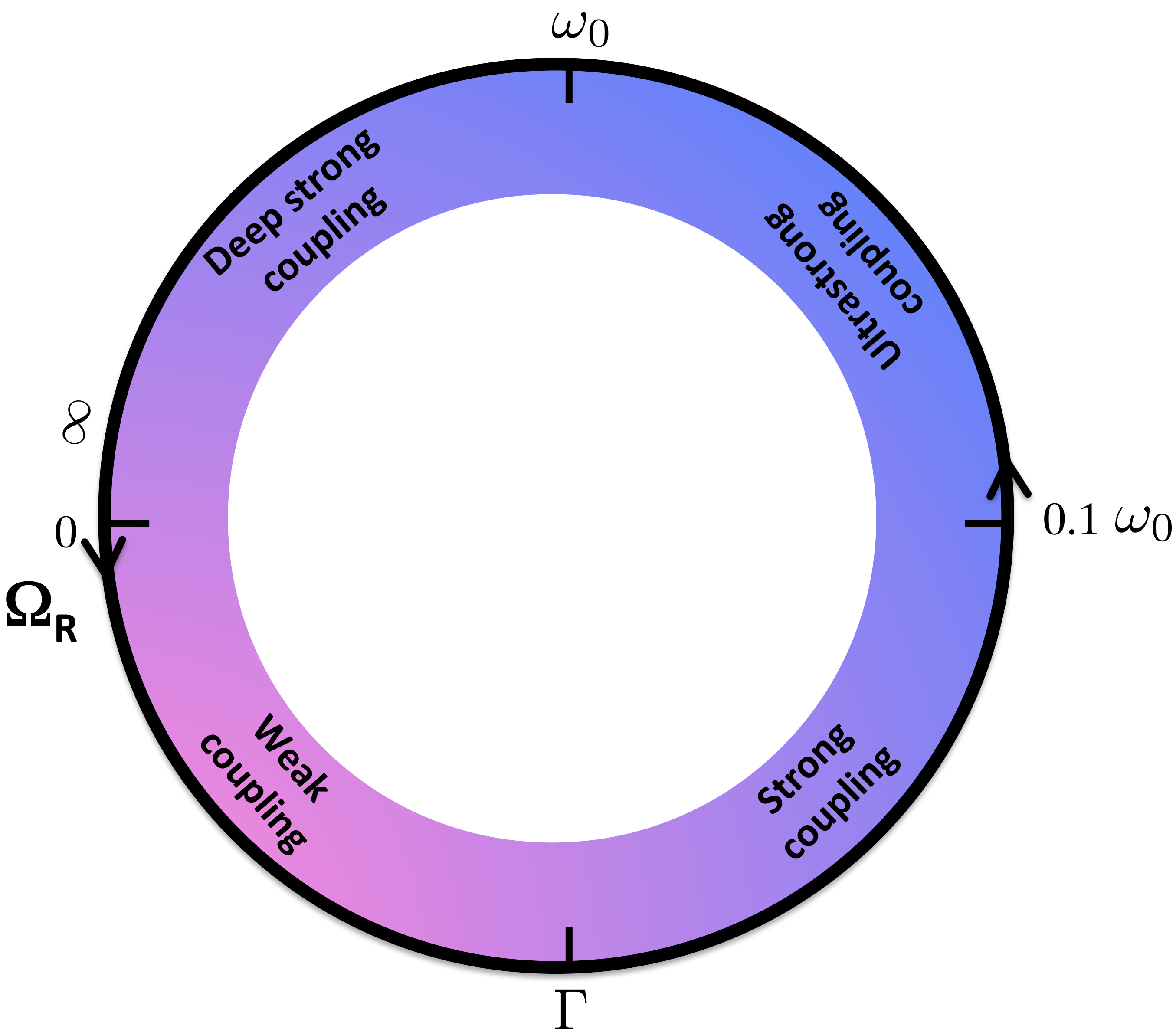
Breakdown of the Purcell effect!

Light-matter coupling



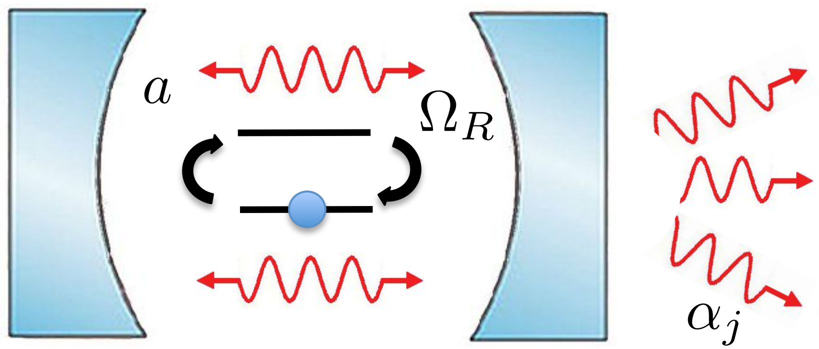
J. Casanova, et al., Phys. Rev. Lett. ***105****, 263603 (2010)*

Light-matter decoupling



Open quantum systems

Open quantum systems



In order to calculate the system emission we have to couple it to the external world

Extra-cavity photonic modes

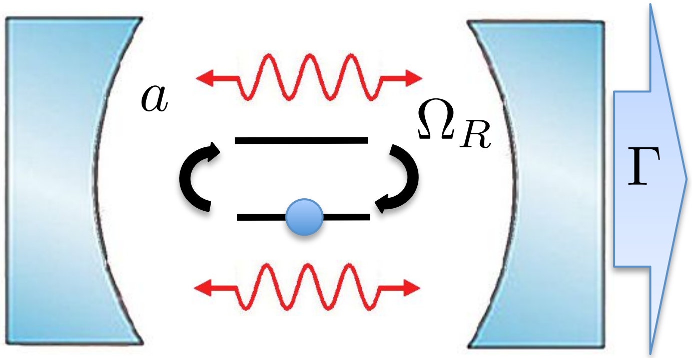
$$H_{\mathrm{SE}} = \sum_j \omega_j \alpha_j^\dagger \alpha_j + \kappa(a^\dagger \alpha_j + \alpha_j^\dagger a)$$

(plus eventually another bath coupled to the matter mode b)

We can then deploy all the arsenal of the theory of open quantum systems

In the ultra and deep strong coupling regime we need to pay attention!

Open quantum systems



The system-environment coupling gives a finite lifetime to cavity photons

$$H_{\mathrm{SE}} = \sum_j \omega_j \alpha_j^\dagger \alpha_j + \kappa(a^\dagger \alpha_j + \alpha_j^\dagger a)$$

Emission rate per photon

Number of emitted photons: $n_{\mathrm{out}} = \Gamma \langle a^\dagger a \rangle$

Number of photons in the cavity

Except that: $\langle G | a^\dagger a | G \rangle = |z_{\mathrm{LP}}|^2 + |z_{\mathrm{UP}}|^2 \neq 0 \propto \frac{\Omega_R^2}{\omega_0^2} + O(\frac{\Omega_R^3}{\omega_0^3})$

Emission of photons out of the ground state. **Wrong!**

Open quantum systems

Master equation: $$\frac{\partial \rho}{\partial t} = -i[H,\, \rho] + \mathcal{L}(\rho)$$

Wrong!

Lindblad operator: $$\mathcal{L}(\rho) = \frac{\kappa}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

Integral Lindbad operator: $$\mathcal{L}(\rho) = \frac{\kappa}{2}(U\rho a^\dagger + a\rho U^\dagger - a^\dagger U\rho - \rho U^\dagger a)$$

$$U = \int_0^\infty dt g(t) e^{-iHt} a e^{iHt}$$

In order to obtain the simplified version: $e^{-iHt} a e^{iHt} \simeq a e^{i\omega_c t}$

Implying: $$U = \int_0^\infty dt g(t) e^{-iHt} a e^{iHt} = \tilde{g}(\omega_c) a$$

False in the ultra and deep strong coupling

S. De Liberato et al, Phys. Rev. A ***80****, 053810 (2009)*
F. Beaudoin, J. M. Gambetta, and A. Blais, Phys. Rev. A ***84****, 043832 (2011)*

Open quantum systems

We considered the photons as fundamental excitations, and coupled them to the external world. But we can also do the opposite.

1) We start from the system-environment Hamiltonian

$$H_{\mathrm{SE}} = \sum_j \omega_j \alpha_j^\dagger \alpha_j + \kappa(a^\dagger \alpha_j + \alpha_j^\dagger a)$$

2) We express the photon operators as a function of the polaritons ones

$$a = x_{\mathrm{LP}} p_{\mathrm{LP}} + x_{\mathrm{UP}} p_{\mathrm{UP}} - z_{\mathrm{LP}} p_{\mathrm{LP}}^\dagger - z_{\mathrm{UP}} p_{\mathrm{UP}}^\dagger$$

3) We obtain a polaritonic system-environment Hamiltonian

$$\begin{aligned} H_{\mathrm{SE}} \quad = \quad & \sum_j \omega_j \alpha_j^\dagger \alpha_j + \kappa(+x_{\mathrm{LP}} \alpha_j p_{\mathrm{LP}}^\dagger + x_{\mathrm{UP}} \alpha_j p_{\mathrm{UP}}^\dagger \\ & -z_{\mathrm{LP}} \alpha_j p_{\mathrm{LP}} - z_{\mathrm{UP}} \alpha_j p_{\mathrm{UP}} + x_{\mathrm{LP}} \alpha_j^\dagger p_{\mathrm{LP}} \\ & +x_{\mathrm{UP}} \alpha_j^\dagger p_{\mathrm{UP}} - z_{\mathrm{LP}} \alpha_j^\dagger p_{\mathrm{LP}}^\dagger - z_{\mathrm{UP}} \alpha_j^\dagger p_{\mathrm{UP}}^\dagger) \end{aligned}$$

Antiresonant term

Resonant term

Open quantum systems

Problem: $H_{\rm SE}|G\rangle_{\rm S} \otimes |0\rangle_{\rm E} \quad \neq \quad 0$

Due to the antiresonant terms in the system-environment coupling the product of the system and environment ground state is not the total ground state

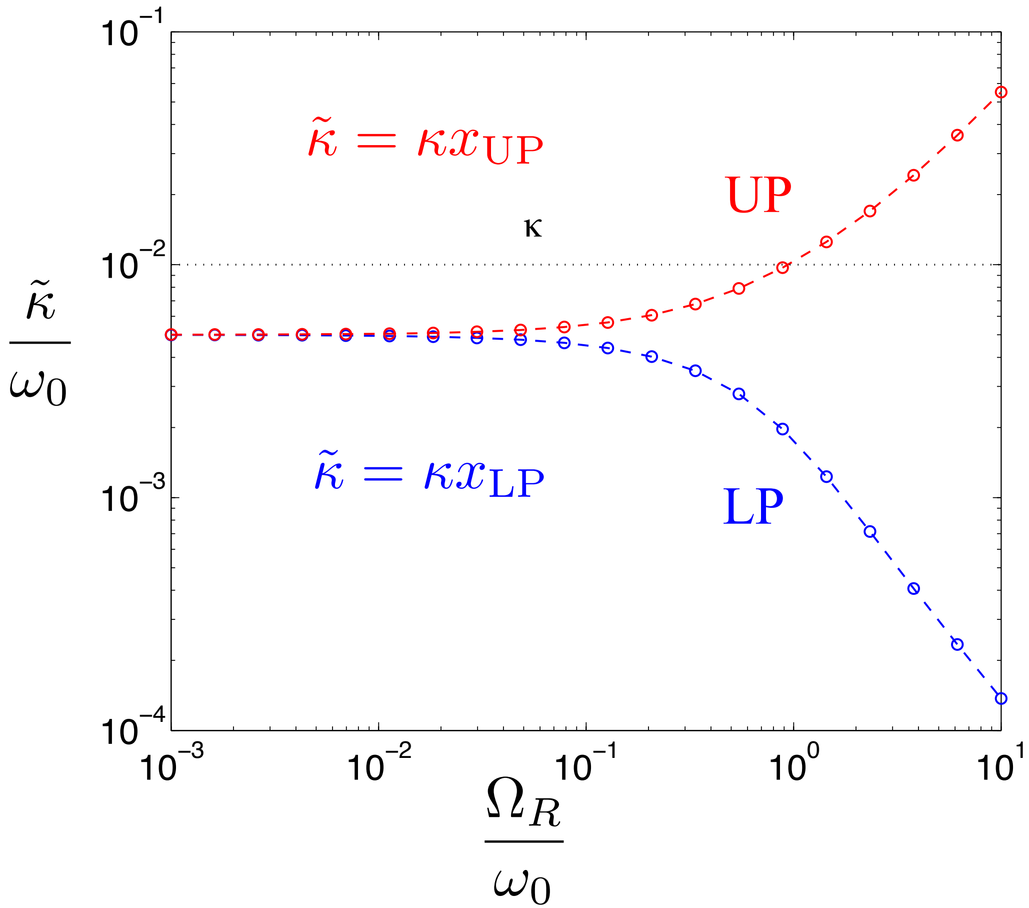
We have to apply the rotating wave approximation

Naïve way

$$
\begin{aligned}
H_{\rm SE} \quad = \quad & \sum_j \omega_j \alpha_j^\dagger \alpha_j + \kappa(+x_{\rm LP}\alpha_j p_{\rm LP}^\dagger + x_{\rm UP}\alpha_j p_{\rm UP}^\dagger \\
& - z_{\rm LP}\cancel{\alpha_j p_{\rm LP}} - z_{\rm UP}\cancel{\alpha_j p_{\rm UP}} + x_{\rm LP}\alpha_j^\dagger p_{\rm LP} \\
& + x_{\rm UP}\alpha_j^\dagger p_{\rm UP} - z_{\rm LP}\cancel{\alpha_j^\dagger p_{\rm LP}^\dagger} - z_{\rm UP}\cancel{\alpha_j^\dagger p_{\rm UP}^\dagger})
\end{aligned}
$$

Open quantum systems

$$H_{\mathrm{SE}}^{\mathrm{RWA}} = \sum_j \omega_j \alpha_j^\dagger \alpha_j + \kappa x_{\mathrm{LP}}(p_{\mathrm{LP}}^\dagger \alpha_j + \alpha_j^\dagger p_{\mathrm{LP}}) + \kappa x_{\mathrm{UP}}(p_{\mathrm{UP}}^\dagger \alpha_j + \alpha_j^\dagger p_{\mathrm{UP}})$$

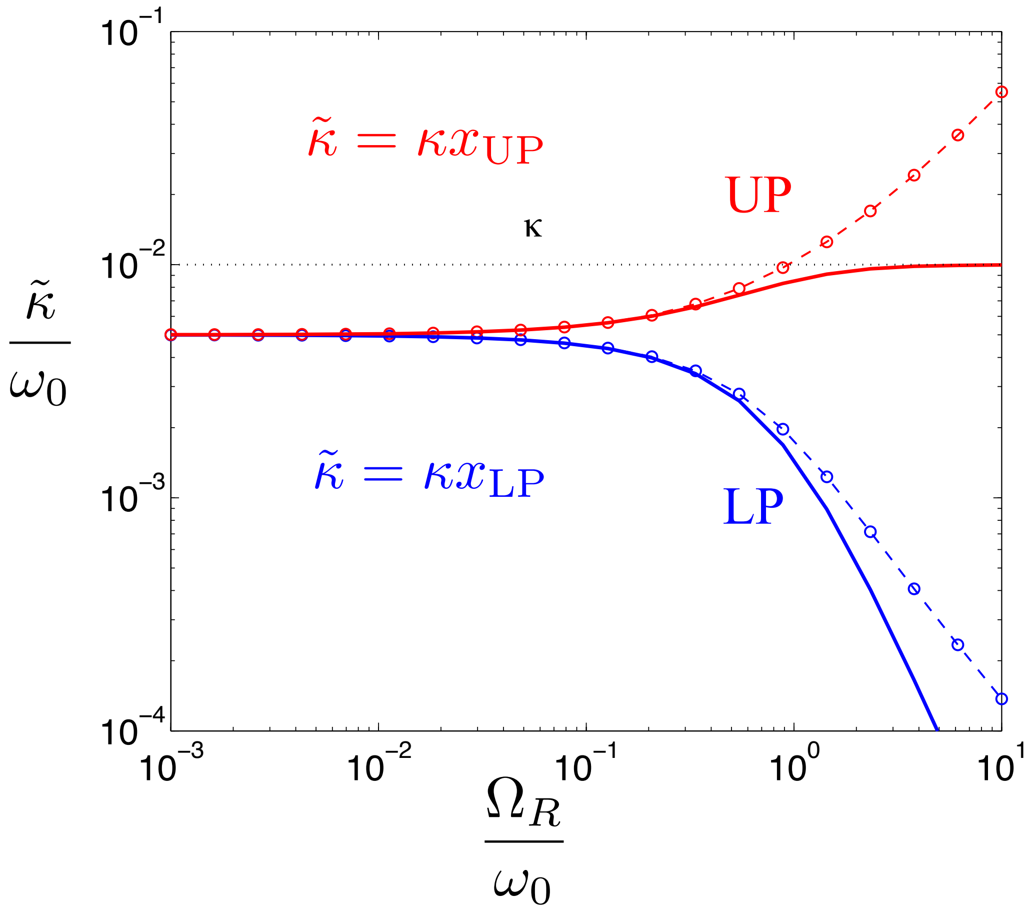


The loss rate of the upper polariton is much larger than the loss rate of a photon.

How can a the coupling with matter increase the mirror losses?

Open quantum systems

$$H_{\mathrm{SE}}^{\mathrm{RWA}} = \sum_j \omega_j \alpha_j^\dagger \alpha_j + \kappa x_{\mathrm{LP}}(p_{\mathrm{LP}}^\dagger \alpha_j + \alpha_j^\dagger p_{\mathrm{LP}}) + \kappa x_{\mathrm{UP}}(p_{\mathrm{UP}}^\dagger \alpha_j + \alpha_j^\dagger p_{\mathrm{UP}})$$



An exact microscopic calculation, using Maxwell boundary conditions gives different results

M. Bamba and T. Ogawa, Phys. Rev. A ***88****,013814 (2013)*

Is the usual open quantum system approach flawed?

Open quantum systems

We are using an Hopfield Bogoliubov transformation
that mixes creation and annihilation operators

$$a = x_{\rm LP} p_{\rm LP} + x_{\rm UP} p_{\rm UP} - z_{\rm LP} p^\dagger_{\rm LP} - z_{\rm UP} p^\dagger_{\rm UP}$$

That needs to be normalised

$$[a,\, a^\dagger] \to |x_{\rm LP}|^2 + |x_{\rm UP}|^2 - |z_{\rm LP}|^2 - |z_{\rm UP}|^2 = 1$$

Doing the rotating wave approximation we are instead considering

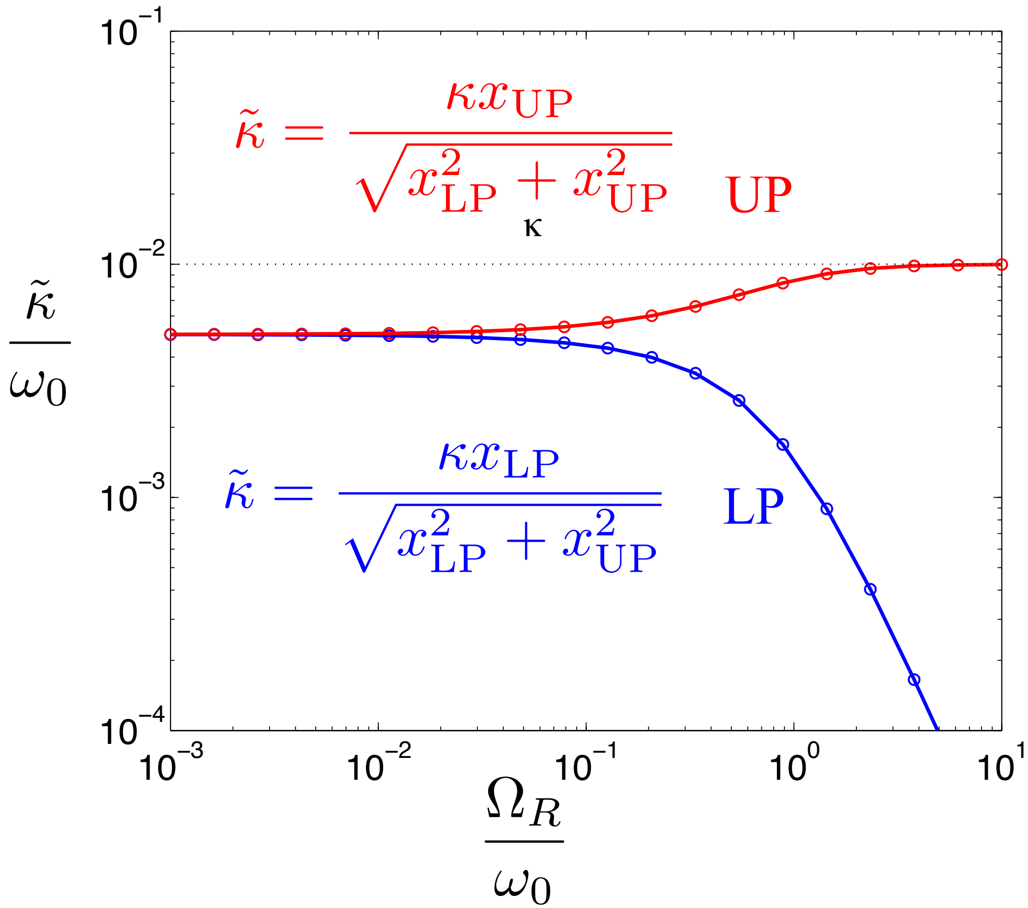
$$|x_{\rm LP}|^2 + |x_{\rm UP}|^2 - \cancel{|z_{\rm LP}|^2} - \cancel{|z_{\rm UP}|^2} > 1$$

The operators are non-normalised

We are removing the non-resonant terms of $H_{\rm SE}$, while we should remove the ones of H . As a consequence $H_{\rm SE}$ would have only resonant terms.

Open quantum systems

$$H_{\mathrm{SE}}^{\mathrm{RWA}} = \sum_j \omega_j \alpha_j^\dagger \alpha_j + \kappa \frac{x_{\mathrm{LP}}}{\sqrt{x_{\mathrm{LP}}^2 + x_{\mathrm{UP}}^2}} (p_{\mathrm{LP}}^\dagger \alpha_j + \alpha_j^\dagger p_{\mathrm{LP}}) + \kappa \frac{x_{\mathrm{UP}}}{\sqrt{x_{\mathrm{LP}}^2 + x_{\mathrm{UP}}^2}} (p_{\mathrm{UP}}^\dagger \alpha_j + \alpha_j^\dagger p_{\mathrm{UP}})$$



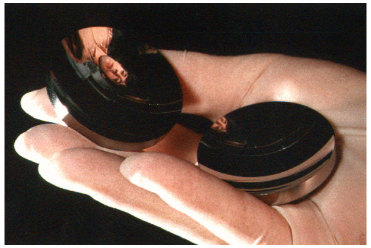
Once the coupling is renormalised, we recover the same result obtained from Maxwell boundary conditions

S. De Liberato, arXiv:1307.5615

Material implementations

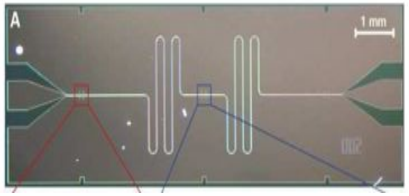
Comparison with other systems

Atoms in superconducting cavities



$$\frac{\Omega_R}{\omega_0} < 0.000001$$

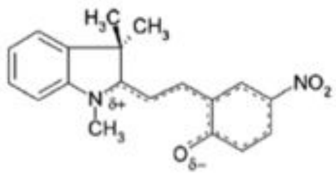
Superconducting circuits (2010)



$$\frac{\Omega_R}{\omega_0} = 0.12$$

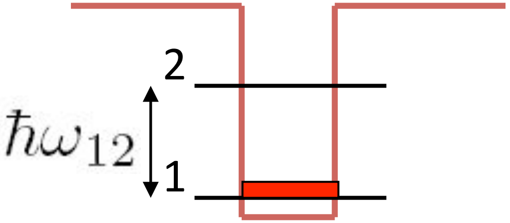
Organic molecules (2011)

Merocyanine



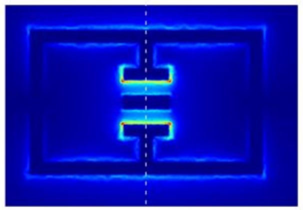
$$\frac{\Omega_R}{\omega_0} = 0.16$$

Intersubband polaritons (2009)



$$\frac{\Omega_R}{\omega_0} = 0.24$$

Landau polaritons (2012)



$$\frac{\Omega_R}{\omega_0} = 0.58$$

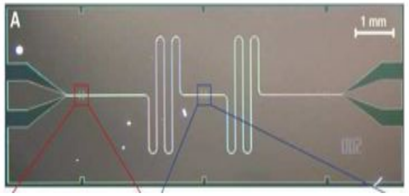
Higher density superradiance
Intrinsically larger dipoles
Better confinement

Comparison with other systems

Atoms in superconducting cavities

$\frac{\Omega_R}{\omega_0} < 0.000001$

Superconducting circuits (2010)



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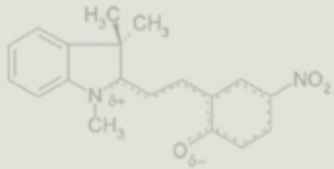

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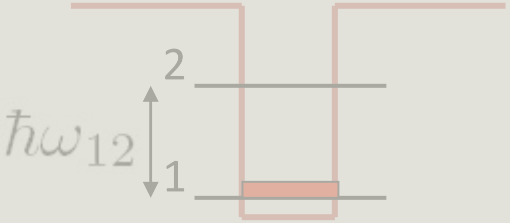

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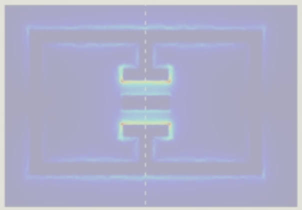

$\frac{\Omega_R}{\omega_0} = 0.58$

Higher density superradiance
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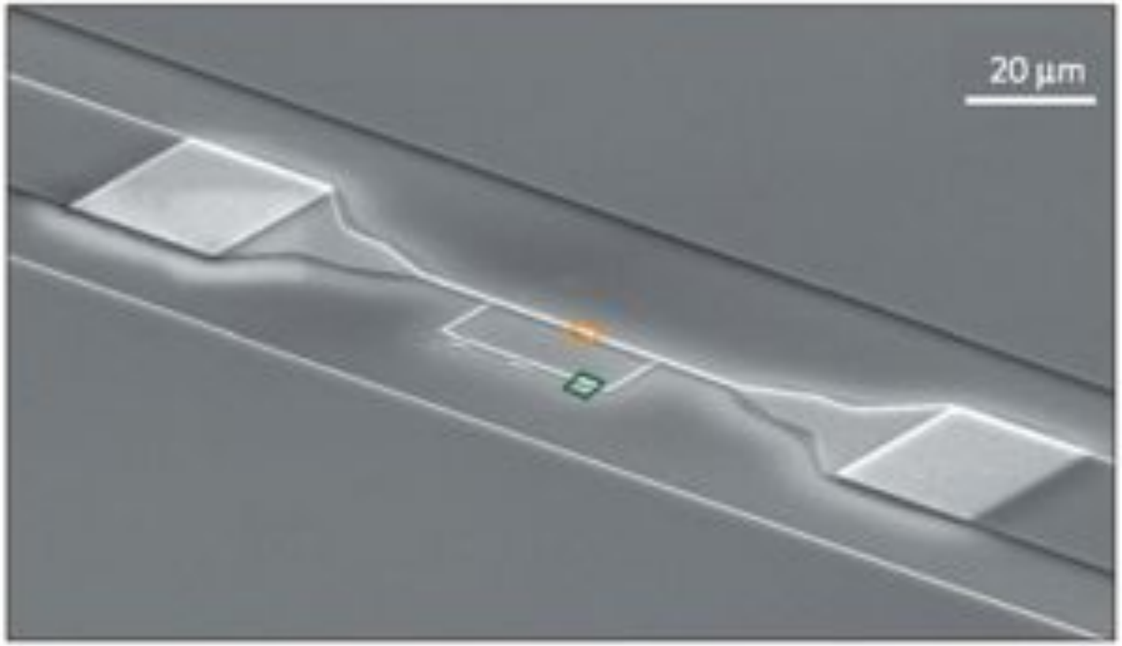
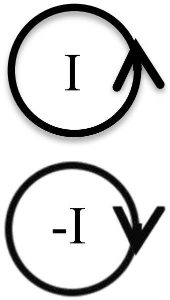
Flux qubit



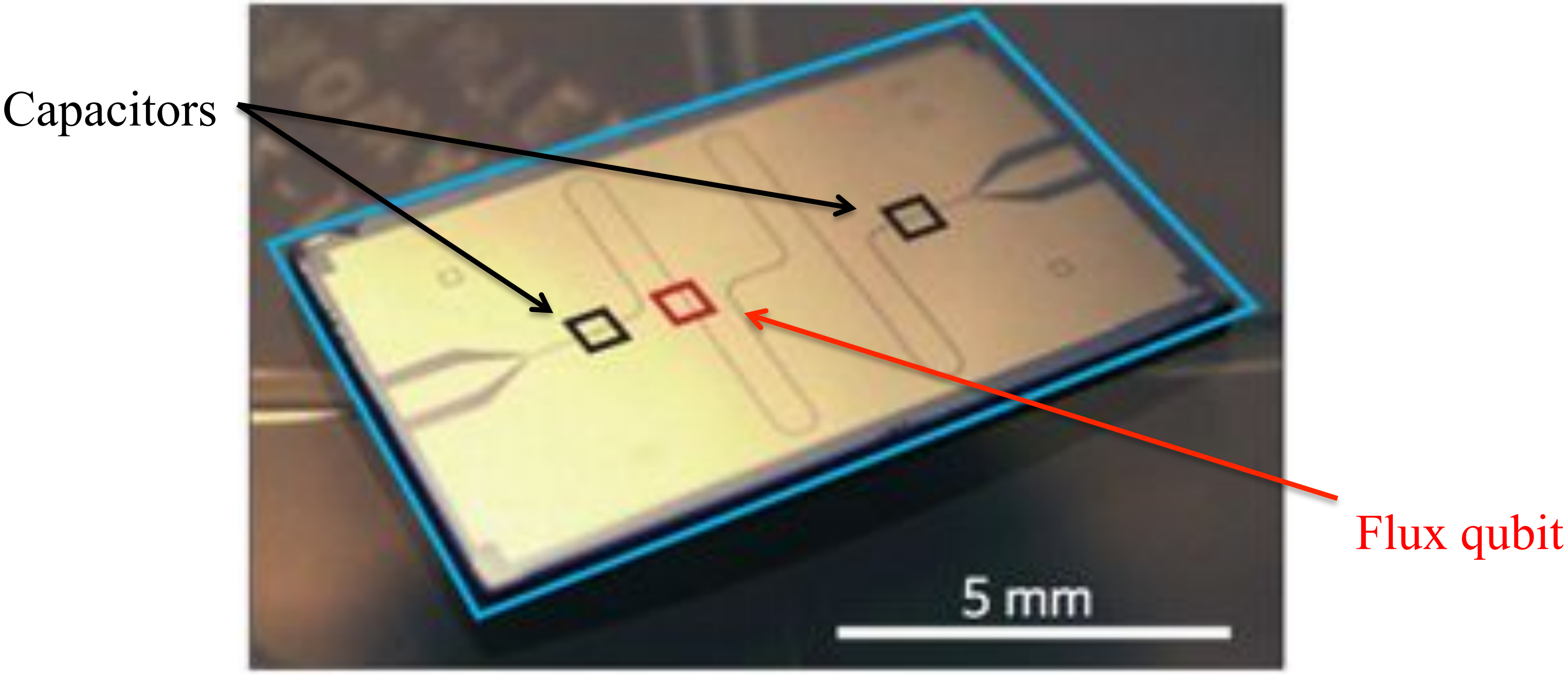
Flux qubit: a superconducting ring in which persistent currents can flow in both directions



$$= |0\rangle$$

$$= |1\rangle$$

Circuit CQED



T. Niemczyk et al., Nat. Phys. ***6****, 772 (2010)*

One single dipole coupled to the electromagnetic field
Jaines-Cummings modes, not Dicke model

$$\frac{\Omega_R}{\omega_0} = 0.12$$

Comparison with other systems

Atoms in superconducting cavities

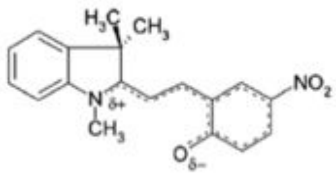
$$\frac{\Omega_R}{\omega_0} < 0.000001$$

Superconducting circuits (2010)

$$\frac{\Omega_R}{\omega_0} = 0.12$$

Organic molecules (2011)

Merocyanine



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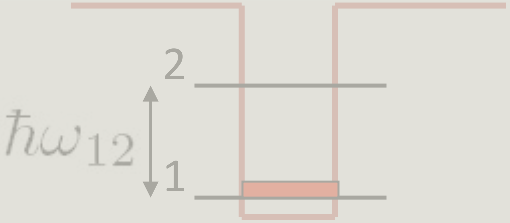

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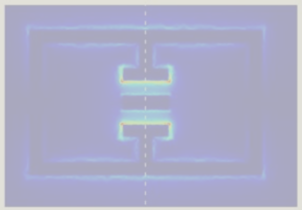
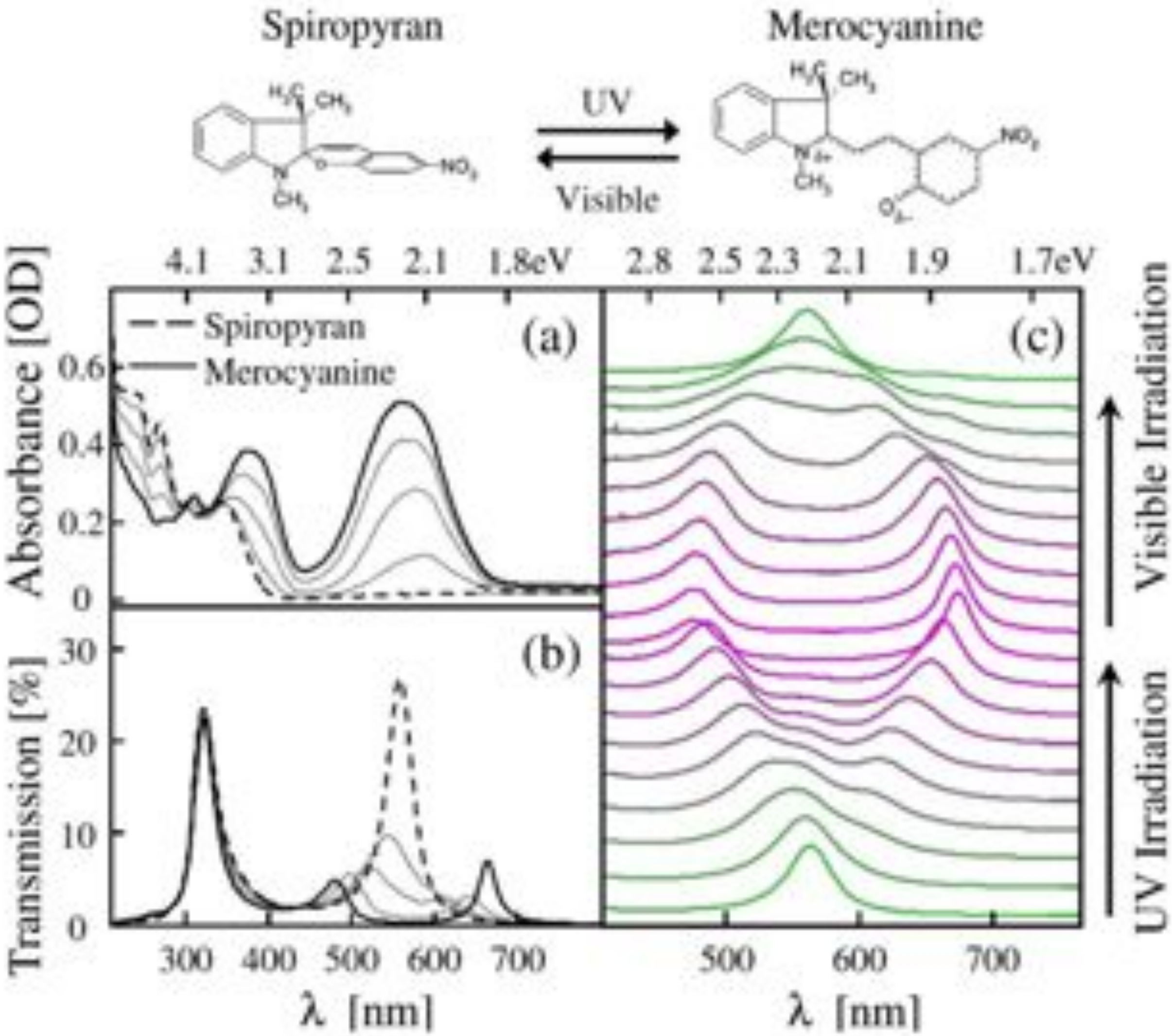
Landau polaritons (2012)



$$\frac{\Omega_R}{\omega_0} = 0.58$$

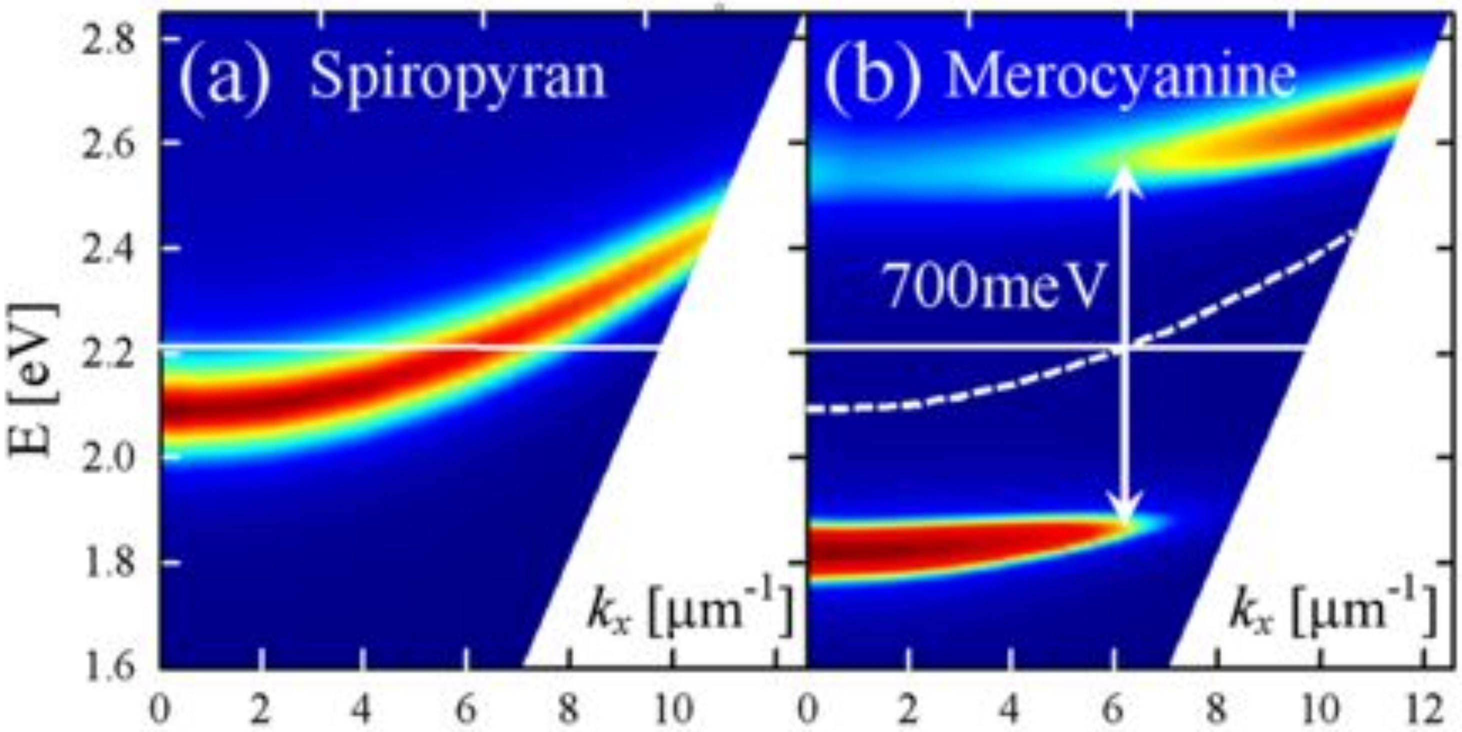
Higher density superradiance
Intrinsically larger dipoles
Better confinement

Switch on



Spiropyran
Merocyanine
UV
Visible
4.1 3.1 2.5 2.1 1.8eV
2.8 2.5 2.3 2.1 1.9 1.7eV
Absorbance [OD]
Spiropyran
Merocyanine
(a)
(b)
(c)
Transmission [%]
λ [nm]
λ [nm]
Visible Irradiation
UV Irradiation

Organic molecules



Largest observed splitting, $\dfrac{\Omega_R}{\omega_0} = 0.16$

T. Schwartz et al., Phys. Rev. Lett. ***106****, 196405 (2011)*

Comparison with other systems

Atoms in superconducting cavities

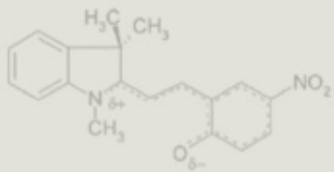
$$\frac{\Omega_R}{\omega_0} < 0.000001$$

Superconducting circuits (2010)

$$\frac{\Omega_R}{\omega_0} = 0.12$$

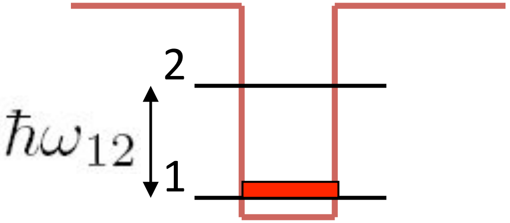
Organic molecules (2011)

Merocyanine



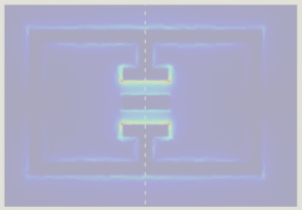
$$\frac{\Omega_R}{\omega_0} = 0.16$$

Intersubband polaritons (2009)



$$\frac{\Omega_R}{\omega_0} = 0.24$$

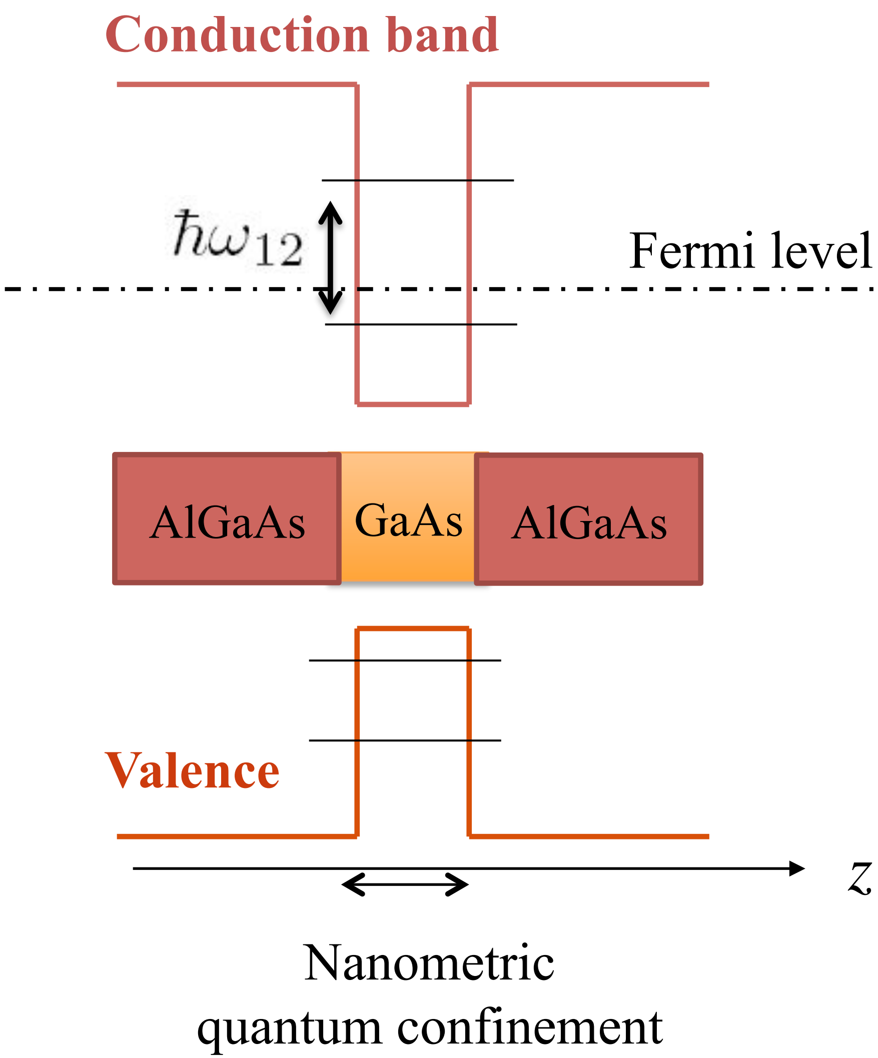
Landau polaritons (2012)

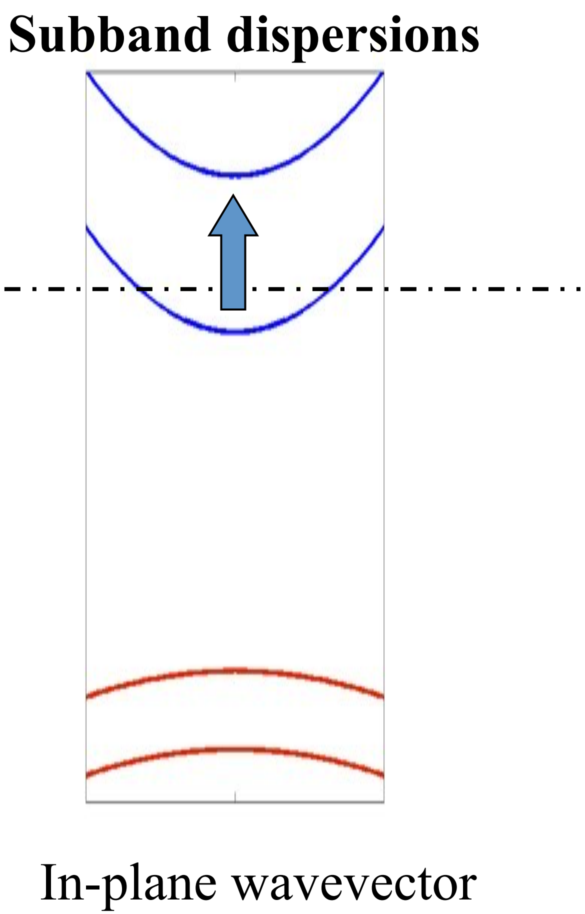


$$\frac{\Omega_R}{\omega_0} = 0.58$$

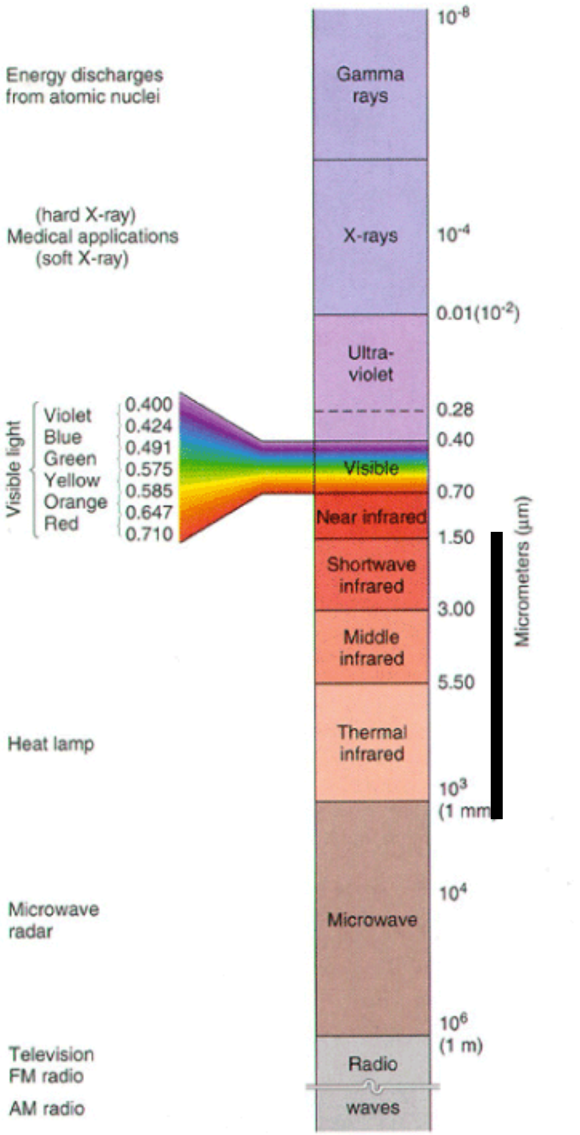
Higher density superradiance
Intrinsically larger dipoles
Better confinement

Doped quantum well

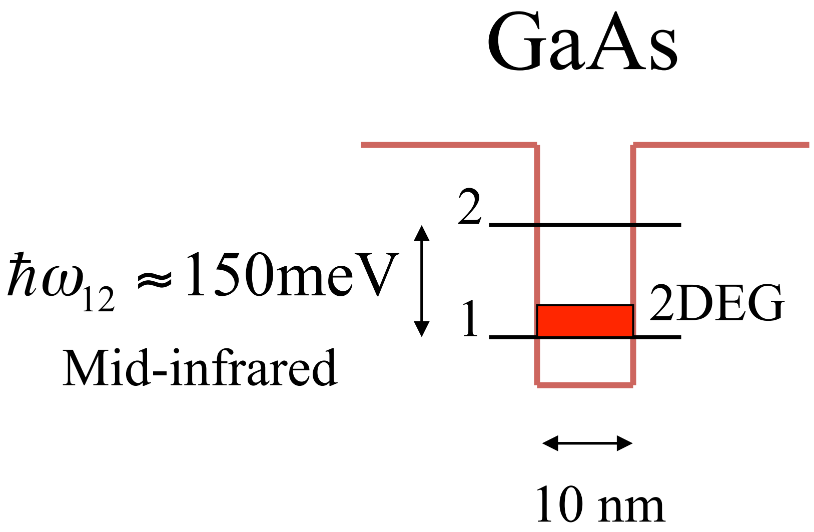




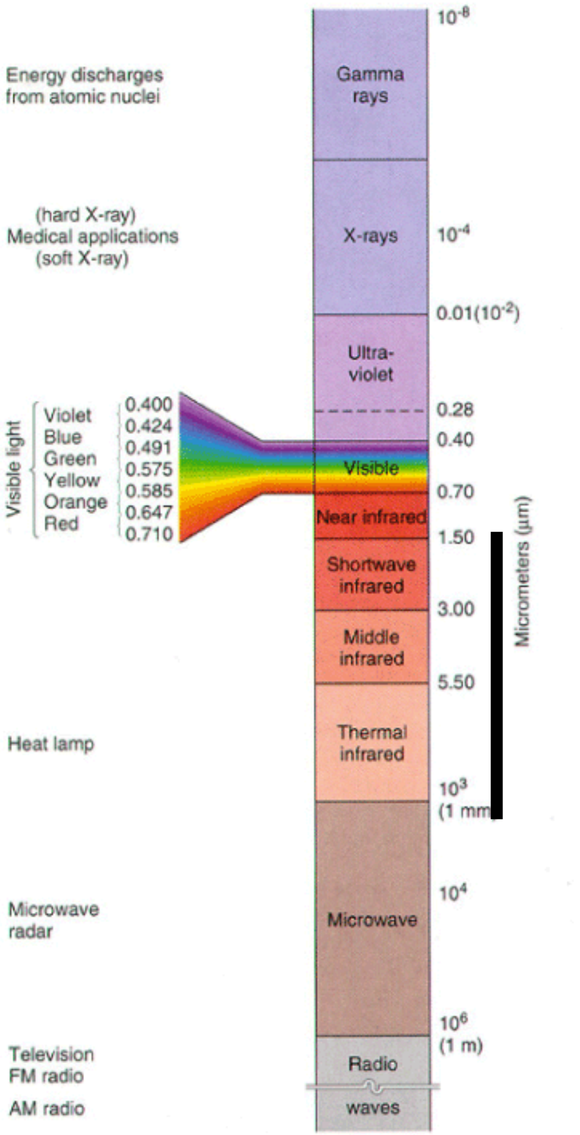
Excitations with Tunable Energy



source: Christopherson (2000) *Geosystems*



Excitations with Tunable Energy



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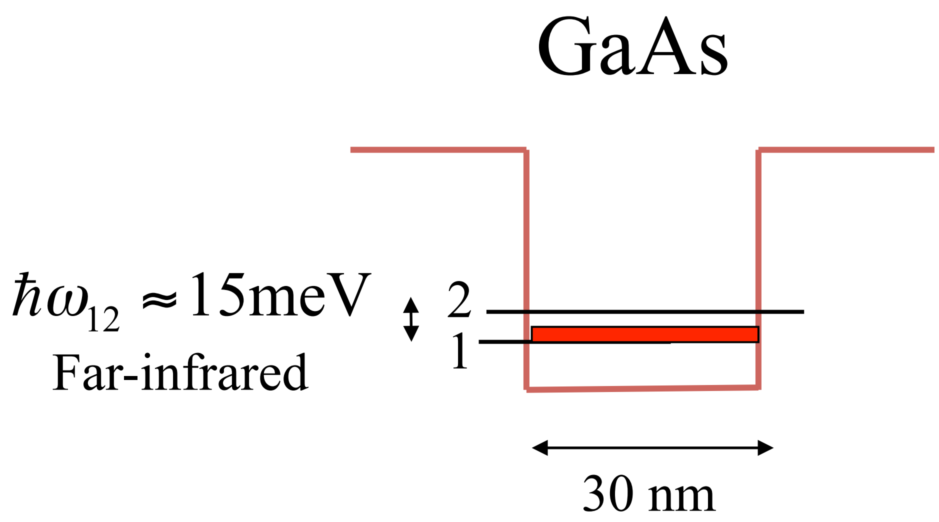
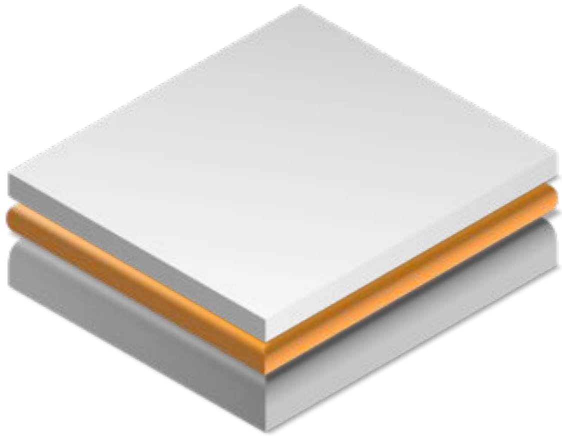


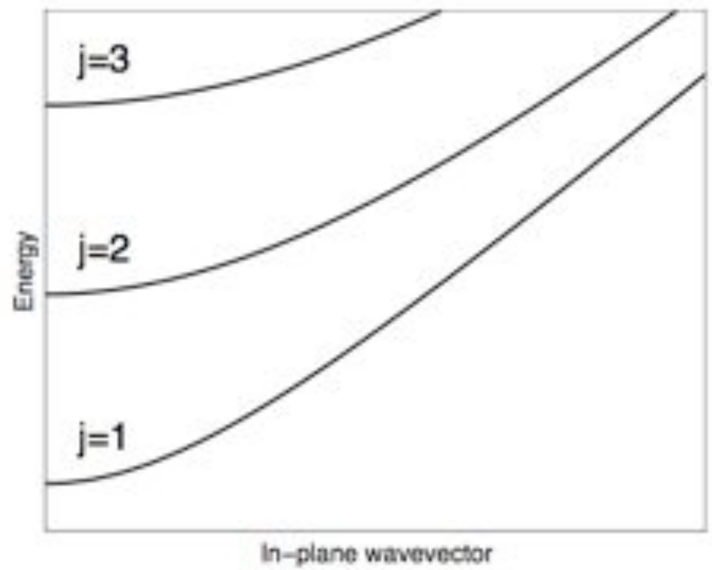
Microcavity



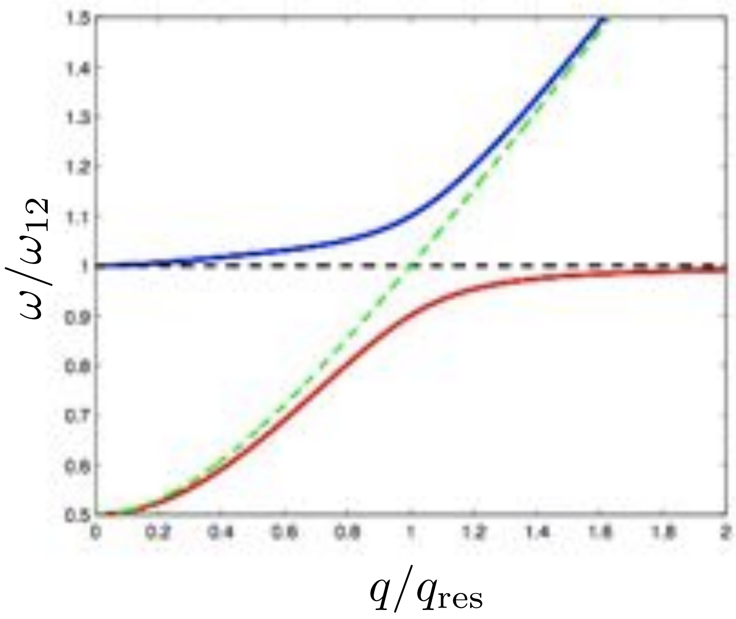
Planar structure that confines photons

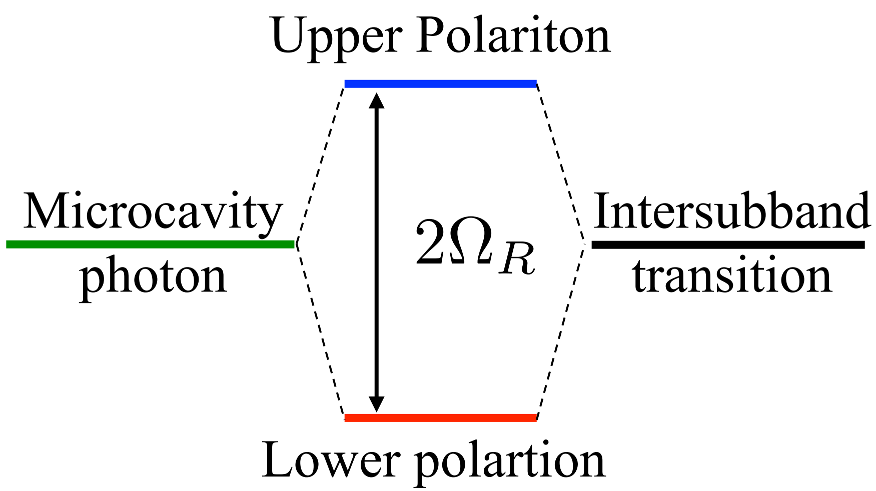
Duble metal configuration

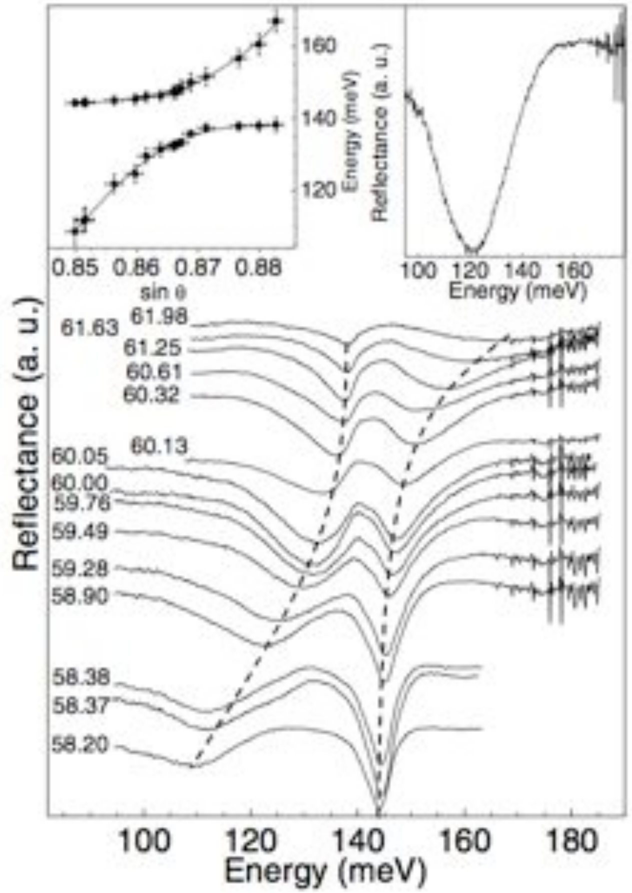
Metal-air configuration



Intersubband polaritons

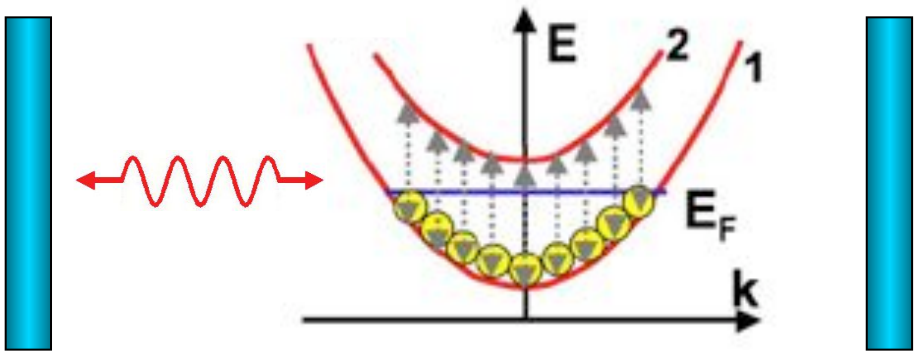






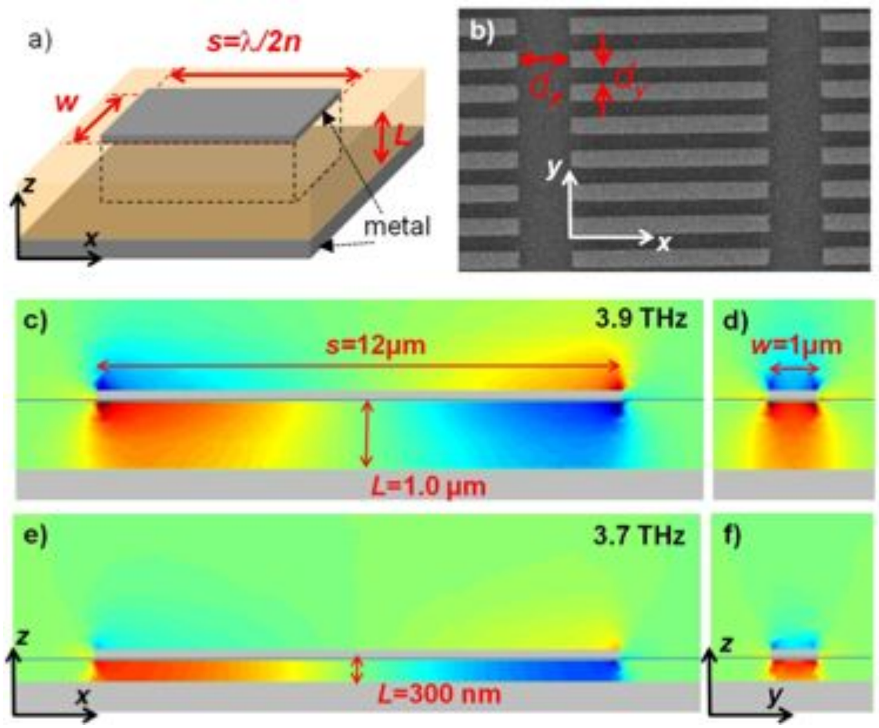
D. Dini et al.,
Phys. Rev. Lett. ***90****, 116401 (2003)*

Enhanced coupling



Superradiant enhancement

$$\Omega_R \propto \sqrt{N_{\mathrm{2DEG}}}$$

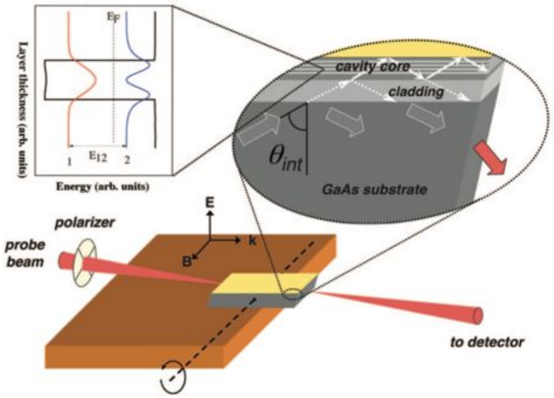


Sub-wavelength confinement

$$\frac{V_{\mathrm{eff}}}{\lambda^3} < 10^{-6}$$

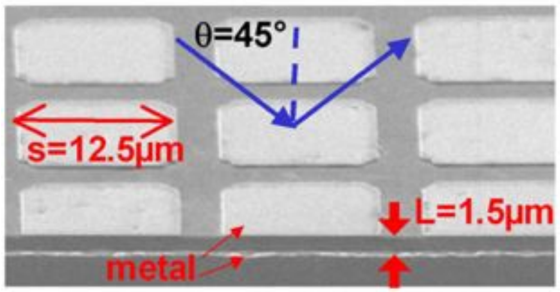
C. Feuillet-Palma et al.,
Opt. Exp. **20**, 29121 (2012).

First observation



A. Anappara et al., Phys. Rev. B ***79****, 201303(R) (2009)*

$$\frac{\Omega_R}{\omega_{12}} = 0.11$$



Y. Todorov et al., Phys. Rev. Lett. **105**, 196402 (2010)

$$\frac{\Omega_R}{\omega_{12}} = 0.24$$

Comparison with other systems

Atoms in superconducting cavities

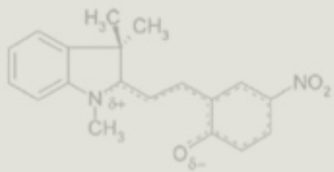
$$\frac{\Omega_R}{\omega_0} < 0.000001$$

Superconducting circuits (2010)

$$\frac{\Omega_R}{\omega_0} = 0.12$$

Organic molecules (2011)

Merocyanine



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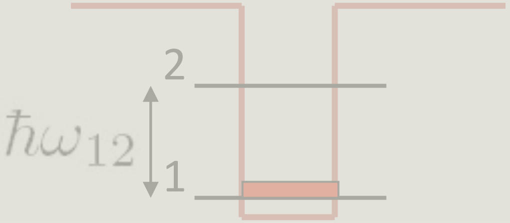
Intersubband polaritons (2009)



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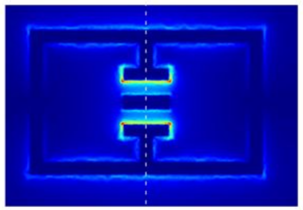
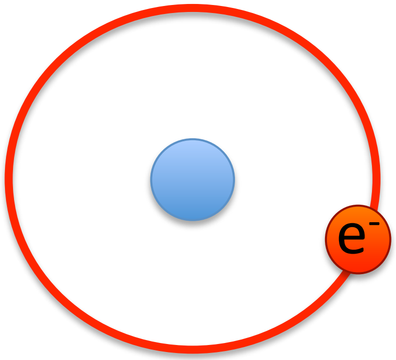


$$\frac{\Omega_R}{\omega_0} = 0.58$$

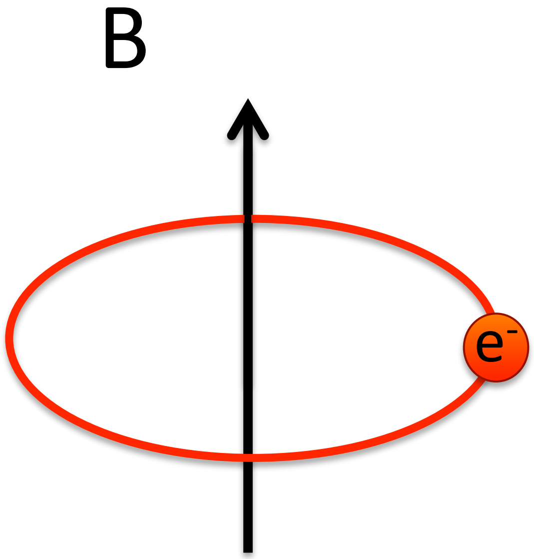
Higher density superradiance
Intrinsically larger dipoles
Better confinement

A naïf idea



Rydberg atom

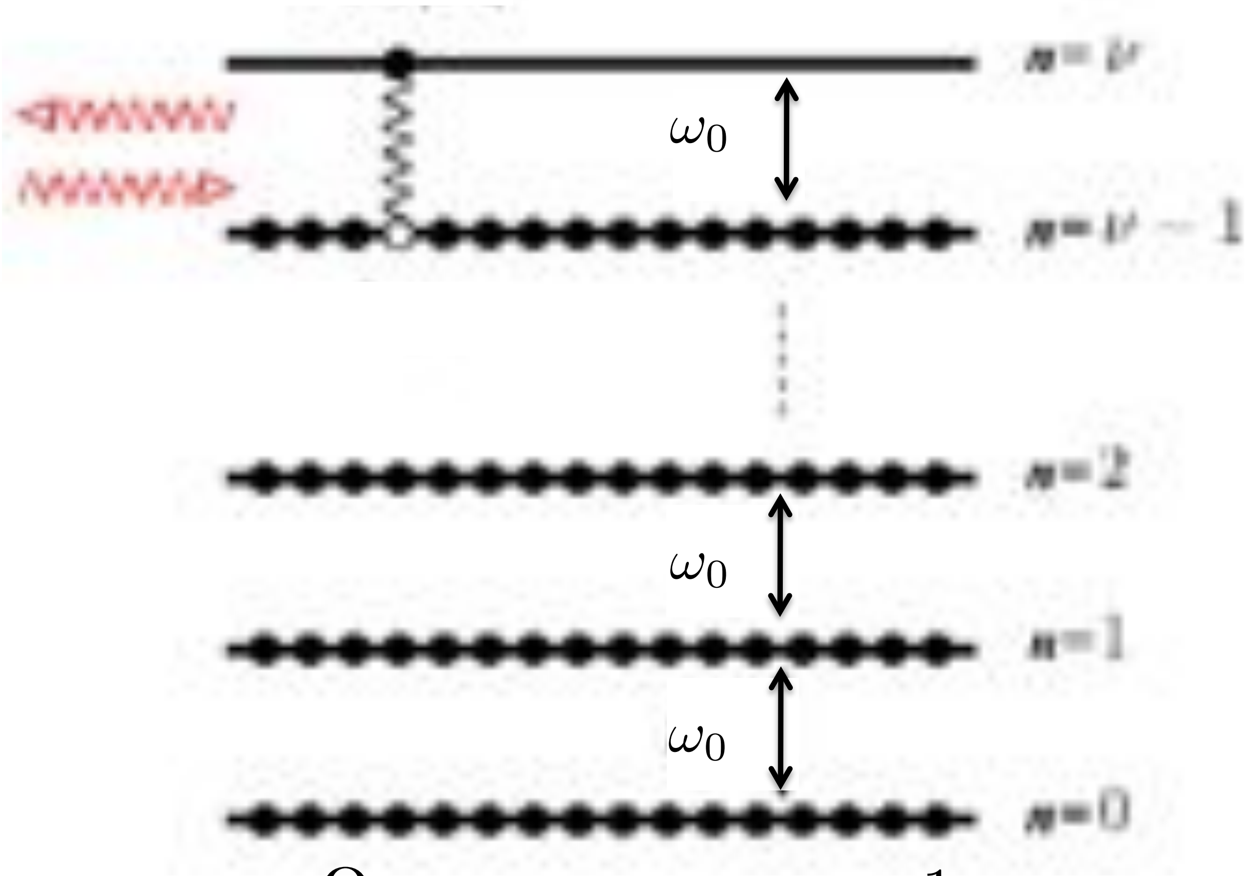
$$r = \frac{n^2 \hbar^2}{e^2 m}$$



Cyclotron orbit

$$r = \frac{mv}{eB}$$

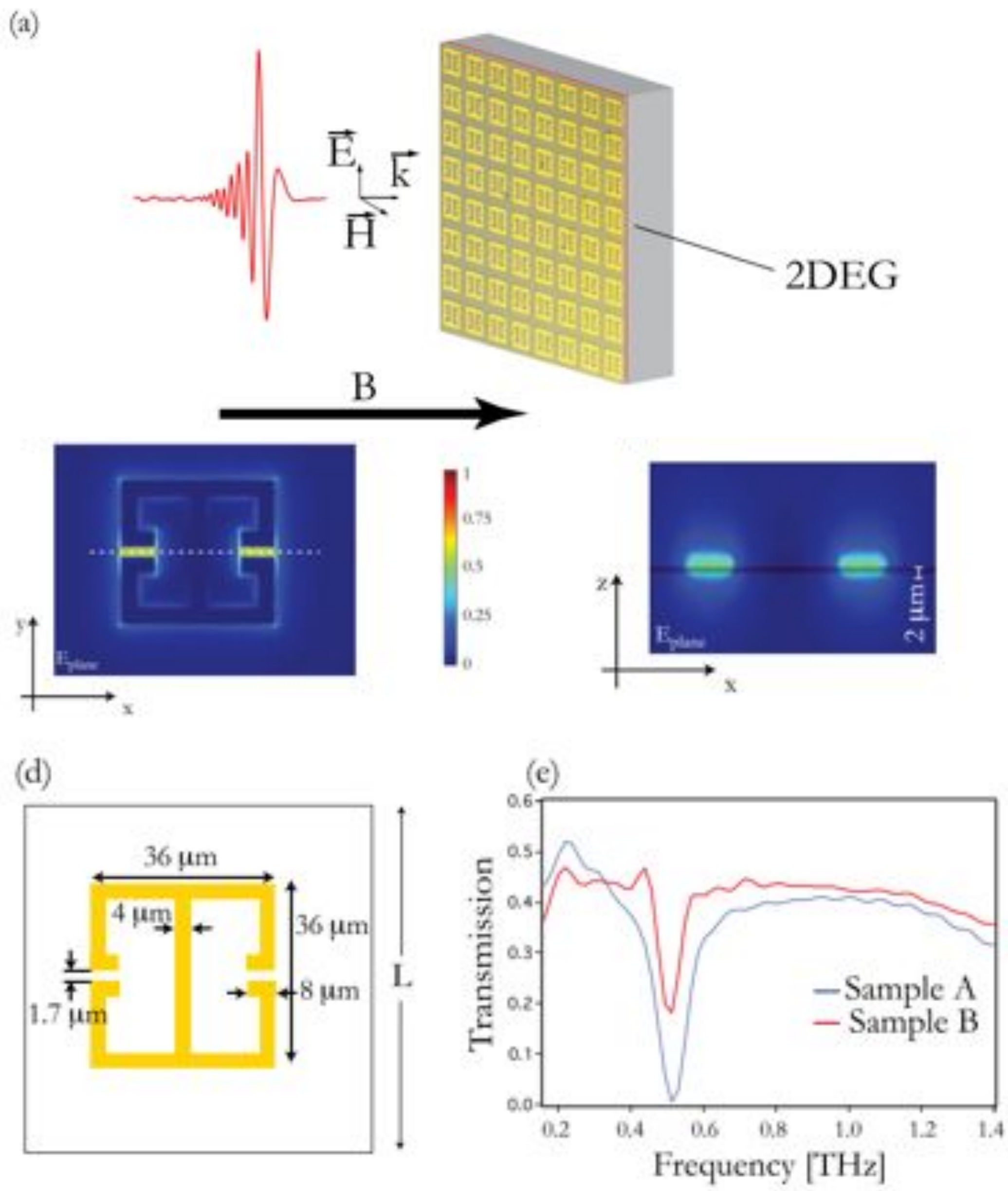
A more realistic description



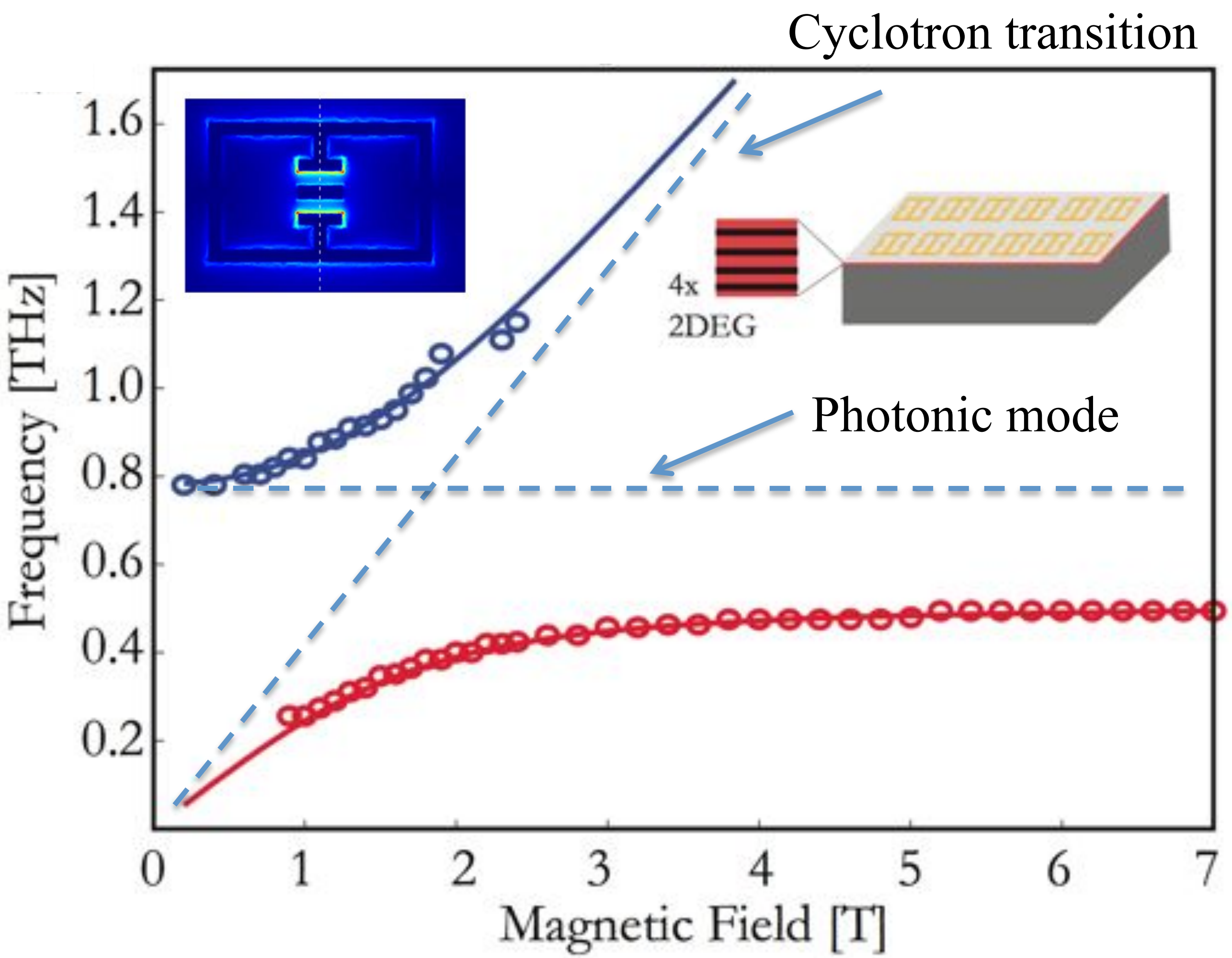
$$\frac{\Omega_R}{\omega_0} \simeq \sqrt{\alpha \nu n_{\mathrm{QW}}} \propto \frac{1}{\sqrt{B}}$$

*D. Hagenmüller, S. De Liberato, and C. Ciuti, PRB **81**, 235303 (2010)*

Sub-wavelength confinement



Experimental observation

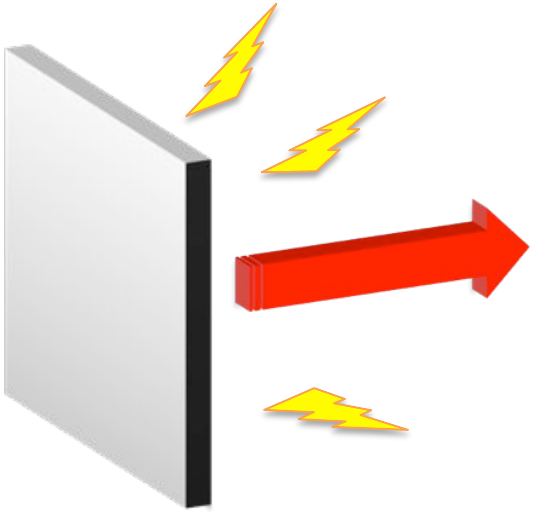


*G. Scalari et al., Science **335**, 1323 (2012)*

$$\frac{\Omega_R}{\omega_0} = 0.58 \quad \nu(1.2T) = 15$$

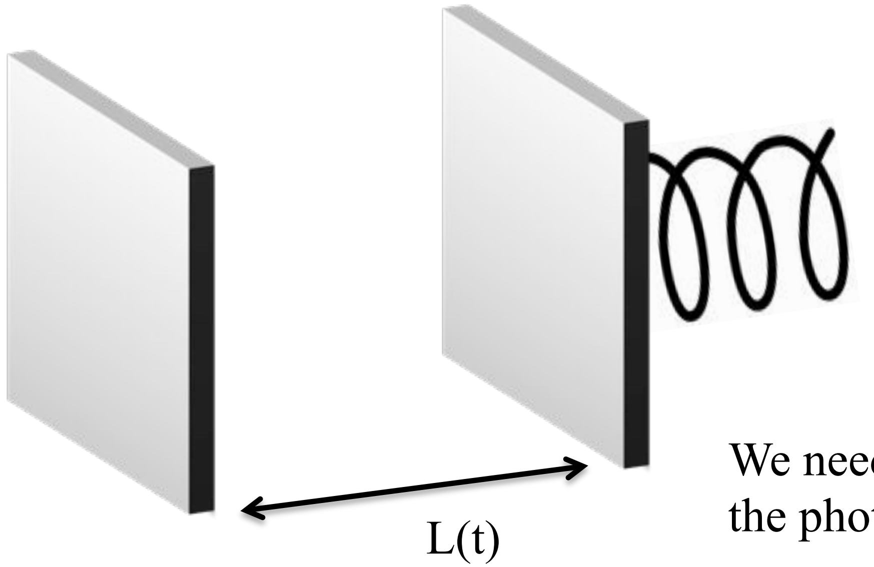
Advanced topics

Dynamical Casimir effect



A mirror accelerated in vacuum emits photons
(due to friction with vacuum fluctuations)

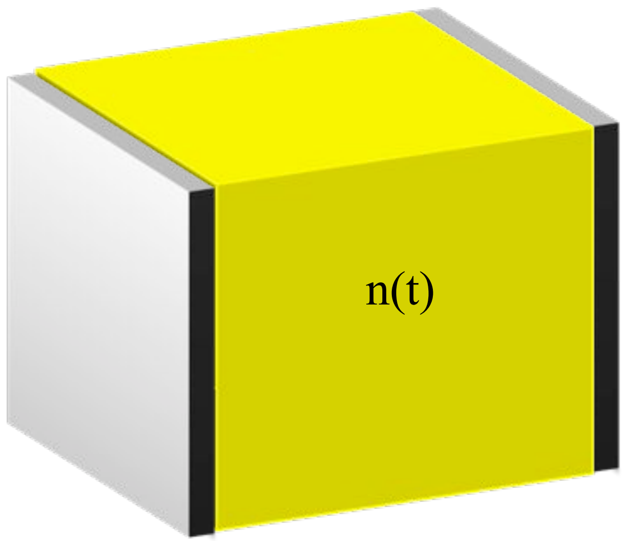
Dynamical Casimir effect



A mirror, accelerated in the vacuum, emits photons

We need the oscillation frequency comparable with the photon one

Impossible for a mechanic spring!



Or, we can keep the length fixed, and change the dielectric constant

$L_{opt}=n(t)L$

No moving parts!

Ultrastrong coupling

The ground state is the state annihilated by the annihilation operators

We call $|0\rangle$ the ground state of the uncoupled light-matter system

$$a|0\rangle = b|0\rangle = 0$$

From the decomposition $p_j = x_j a + y_j b + z_j a^\dagger + w_j b^\dagger$

$$p_j|0\rangle \neq 0$$

The coupling modifies the ground state

We introduce the ground state of the coupled system $|G\rangle$

$$p_j|G\rangle = 0$$

We have then $\langle G|a^\dagger a|G\rangle = |z_{\mathrm{LP}}|^2 + |z_{\mathrm{UP}}|^2 \neq 0 \propto \frac{\Omega_R^2}{\omega_0^2} + O(\frac{\Omega_R^3}{\omega_0^3})$

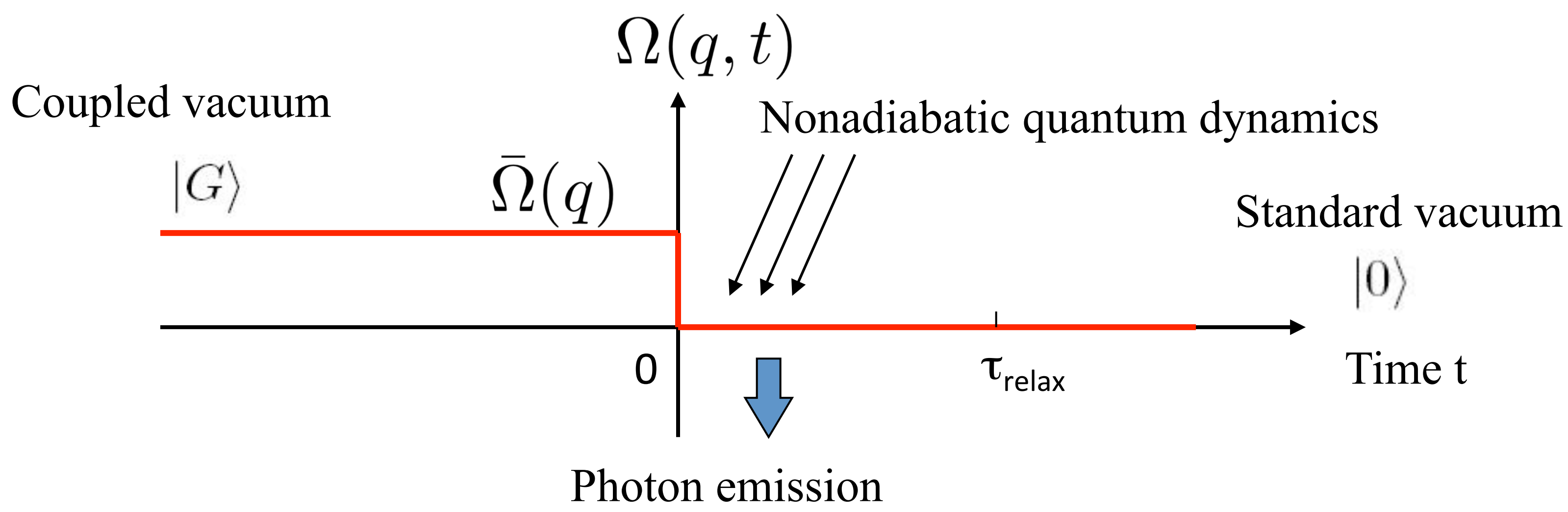
The ground state contains a population of bound photons

Quantum vacuum emission

The coupling changes the ground state

Free system: $|0\rangle$ ← Standard vacuum

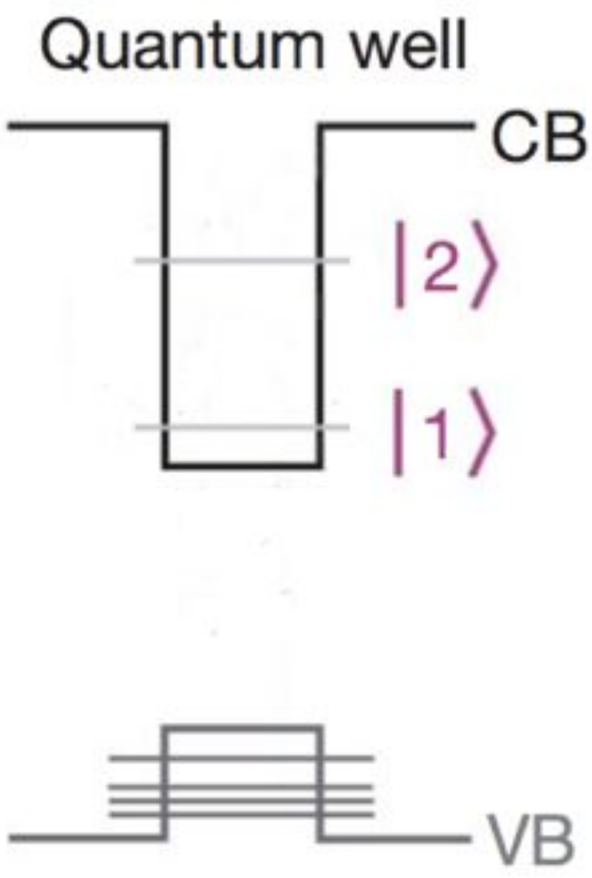
Coupled oscillators: $|G\rangle$ ← Coupled vacuum

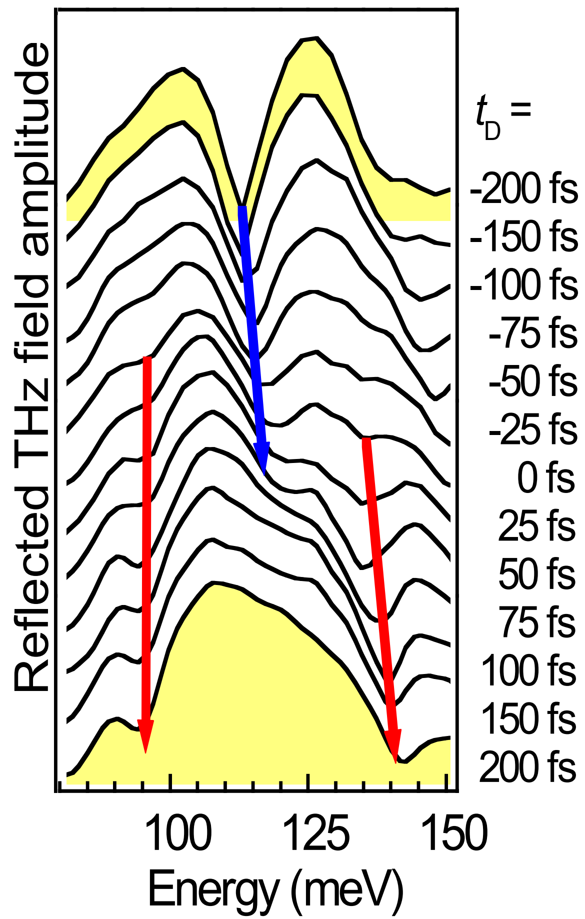


$$\langle G|a^\dagger a|G\rangle = |z_1|^2 + |z_2|^2 \propto \frac{\Omega^2}{\omega^2} + O\left(\frac{\Omega^3}{\omega^3}\right)$$

S. De Liberato, C. Ciuti and I. Carusotto, Phys. Rev. Lett. ***98****, 103602 (2007)*

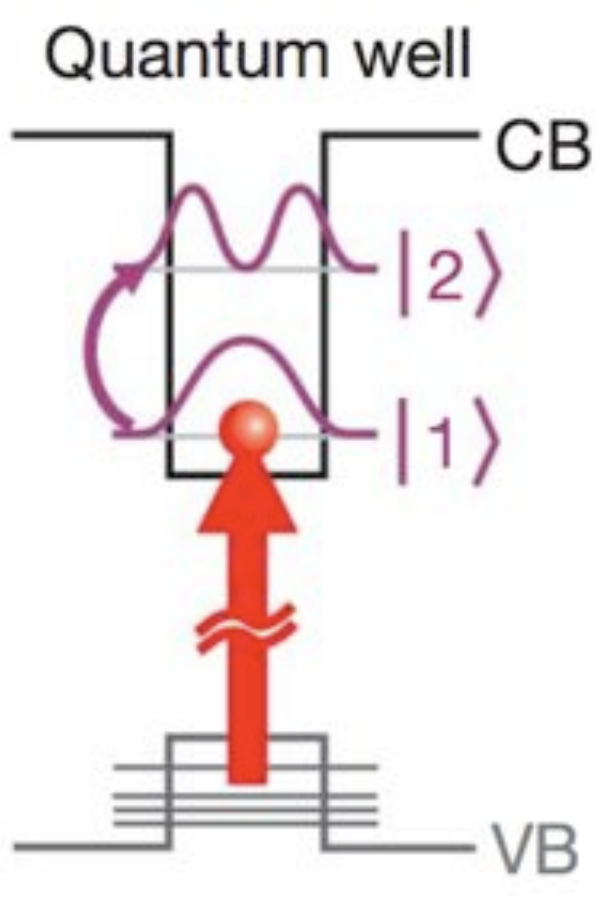
Nonadiabatic modulation

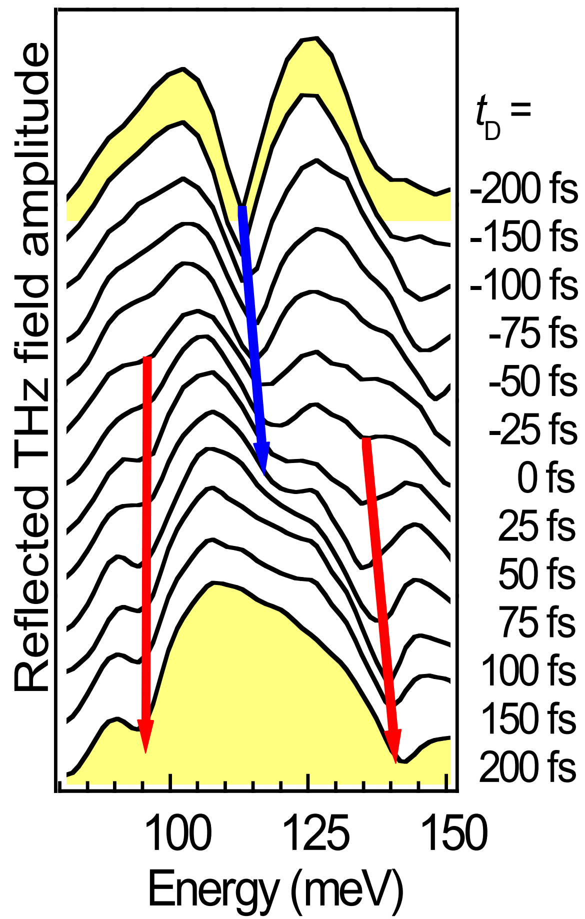




G. Guenter et al., Nature ***458****, 178 (2009)*

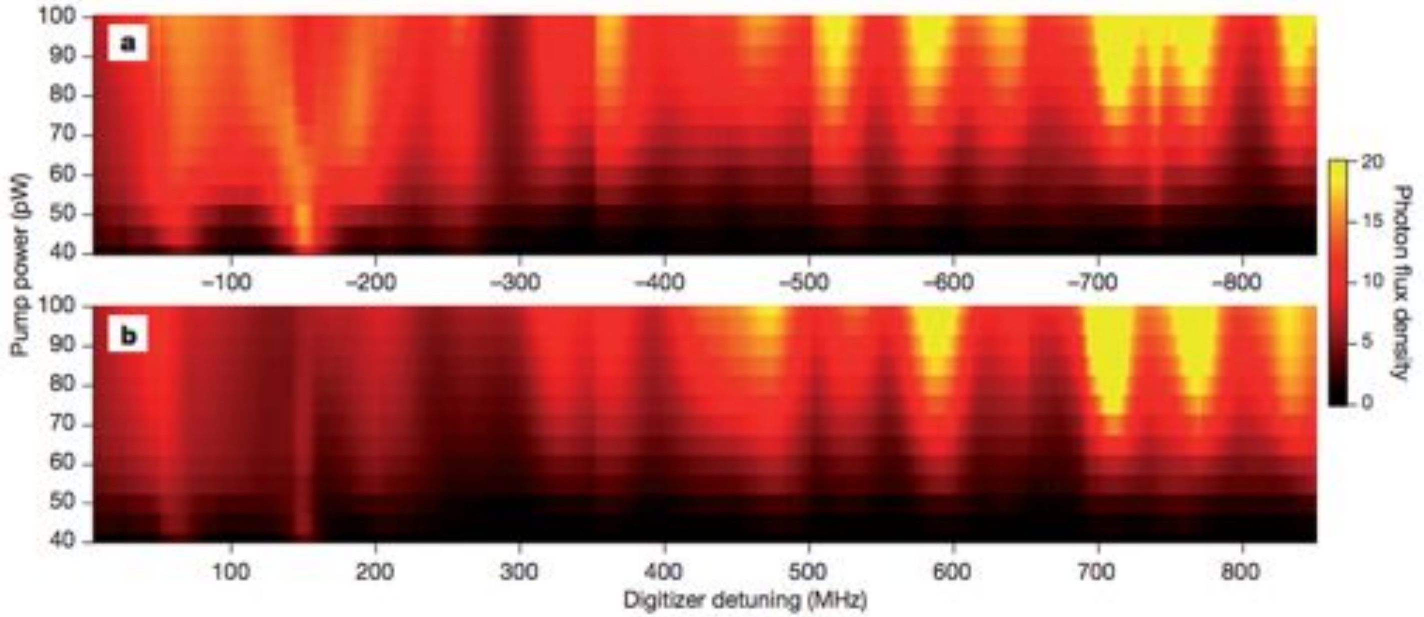
Nonadiabatic modulation





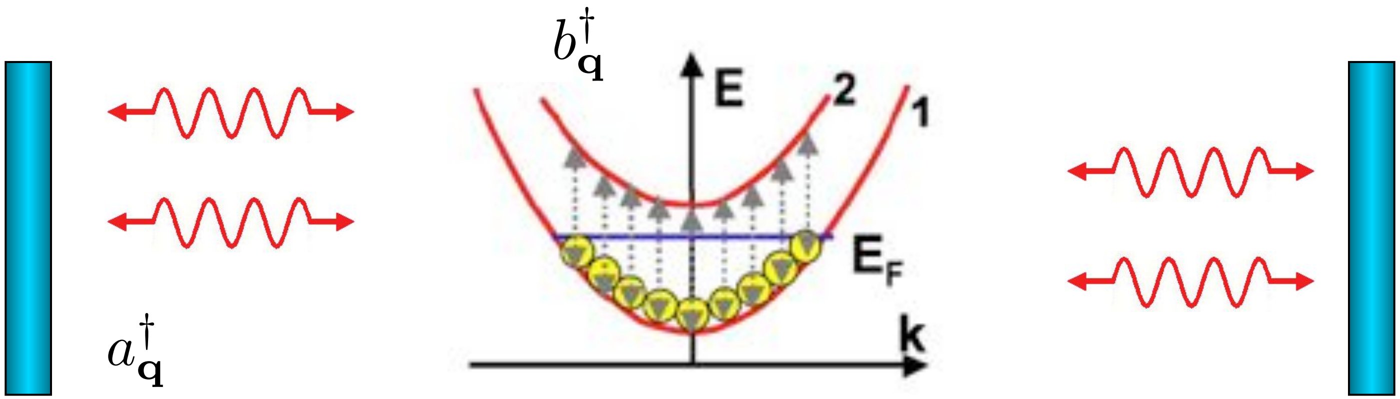
*G. Guenter et al., Nature **458**, 178 (2009)*

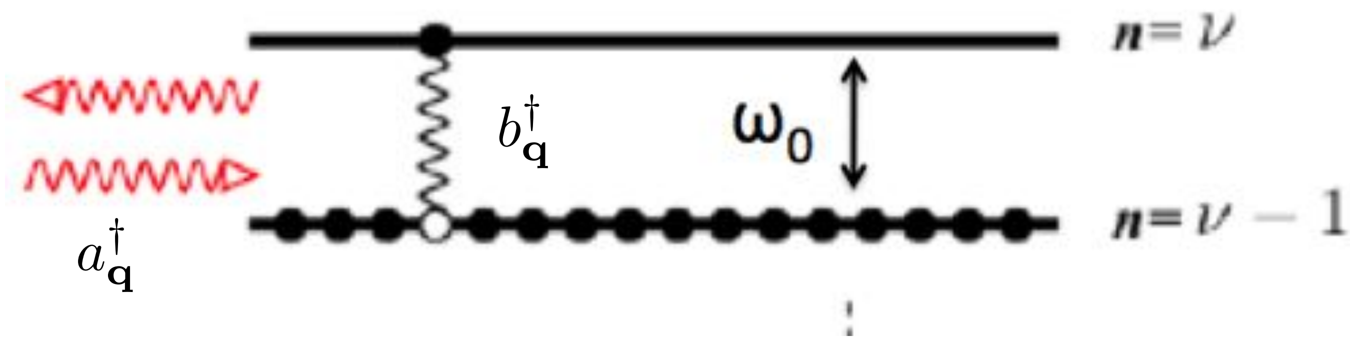
First observation



C. M. Wilson et al., Nature ***479****, 376 (2011)*

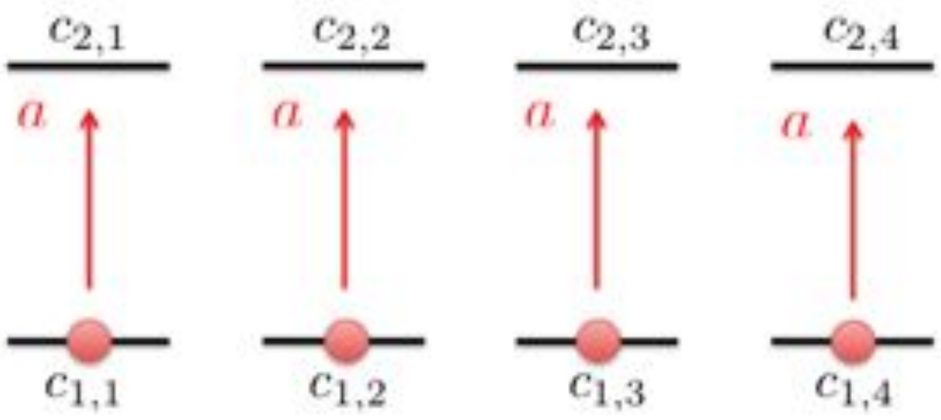
Beyond the Dicke model



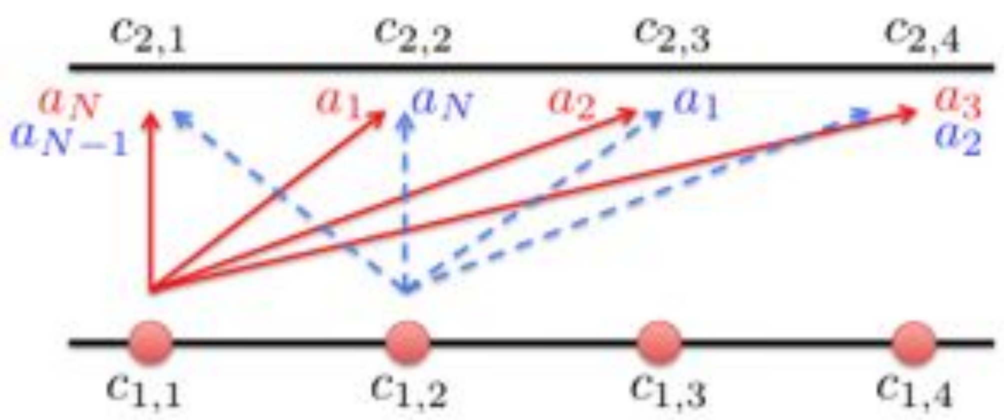


These systems are modeled as Dicke models.

The Hilbert space is a large place



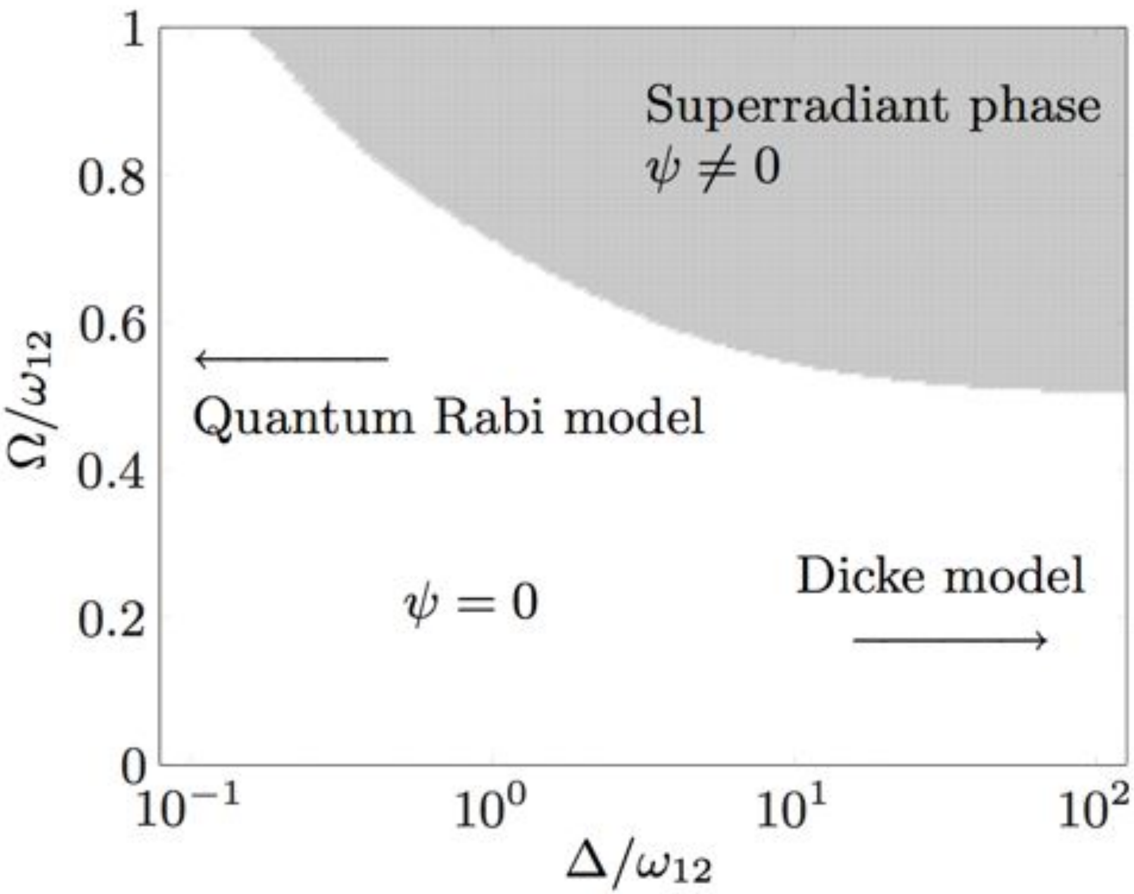
$$d_{\text{Dicke}} = 2^N$$



$$d_{TBM} = \begin{pmatrix} 2N \\ N \end{pmatrix} \rightarrow \frac{4^N}{\sqrt{N\pi}}$$

A flat band model is not a Dicke model!
One mode approximation is justified only in the linear regime!

Quantum Phase Transitions



Toward flat photonic dispersions
Many modes resonant with the transition

Toward steep photonic dispersions
Few modes resonant with the transition

S. De Liberato and C. Ciuti, Phys. Rev. Lett. ***110****, 133603 (2013)*

Thank you for your attention