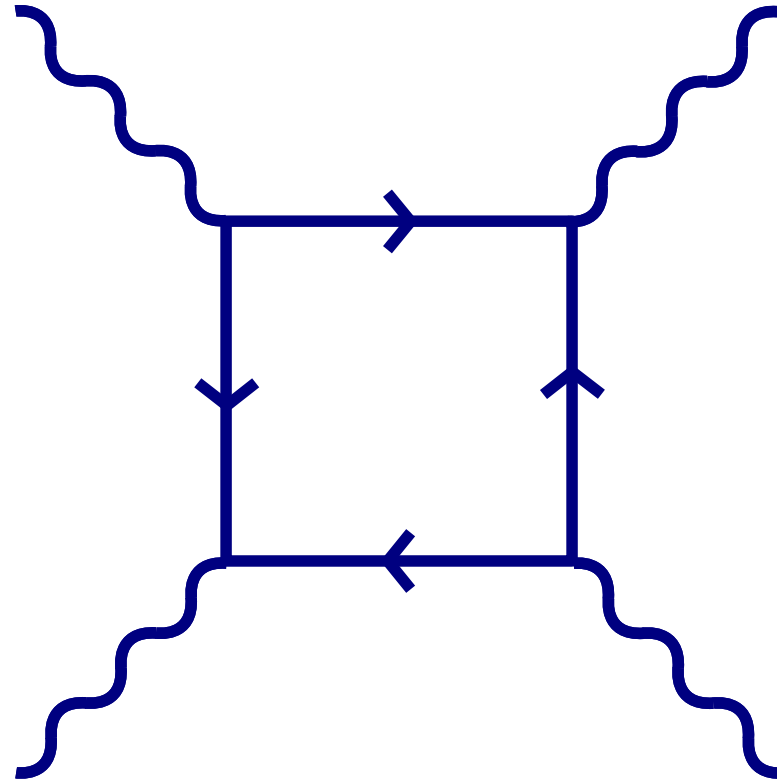


Strongly interacting and strongly correlated photons in strongly nonlinear cavities

Iacopo Carusotto

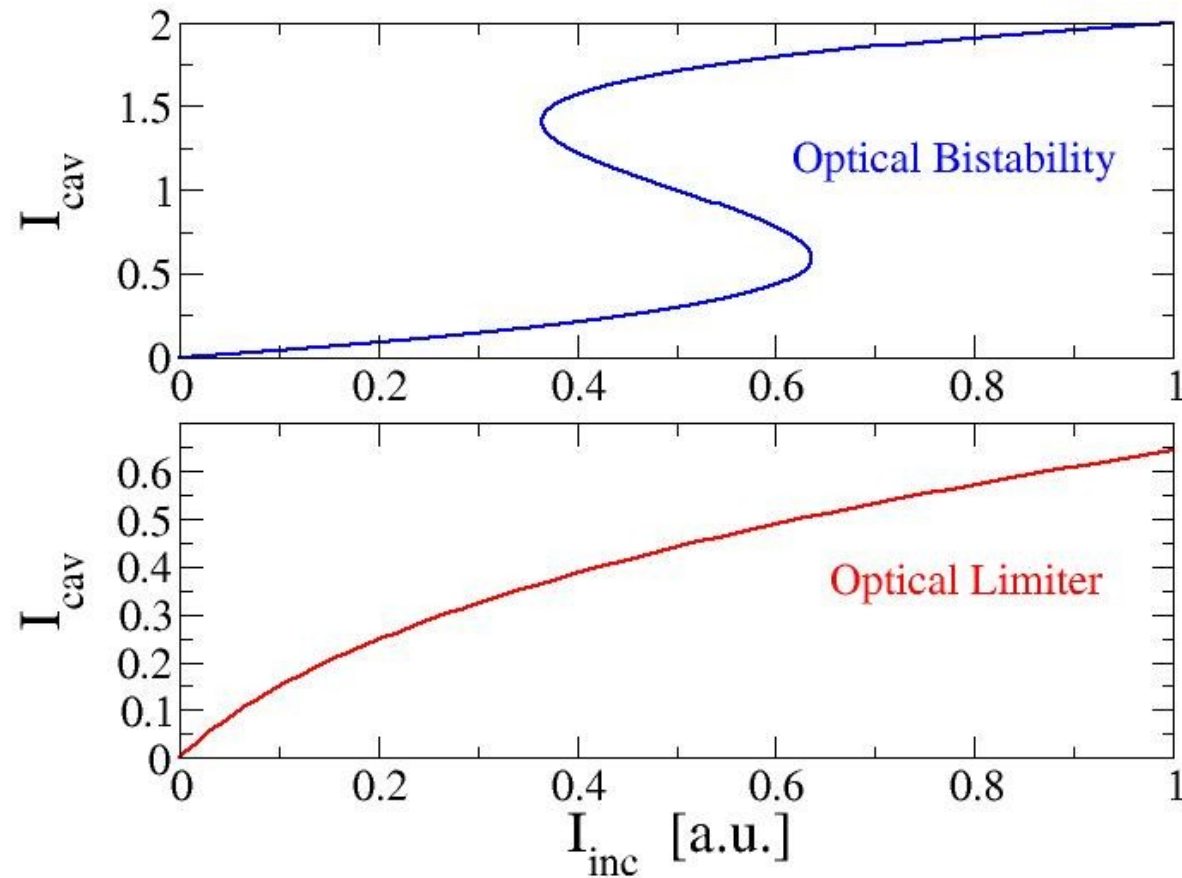
INO-CNR BEC Center and Università di Trento, Italy

Photon-photon scattering

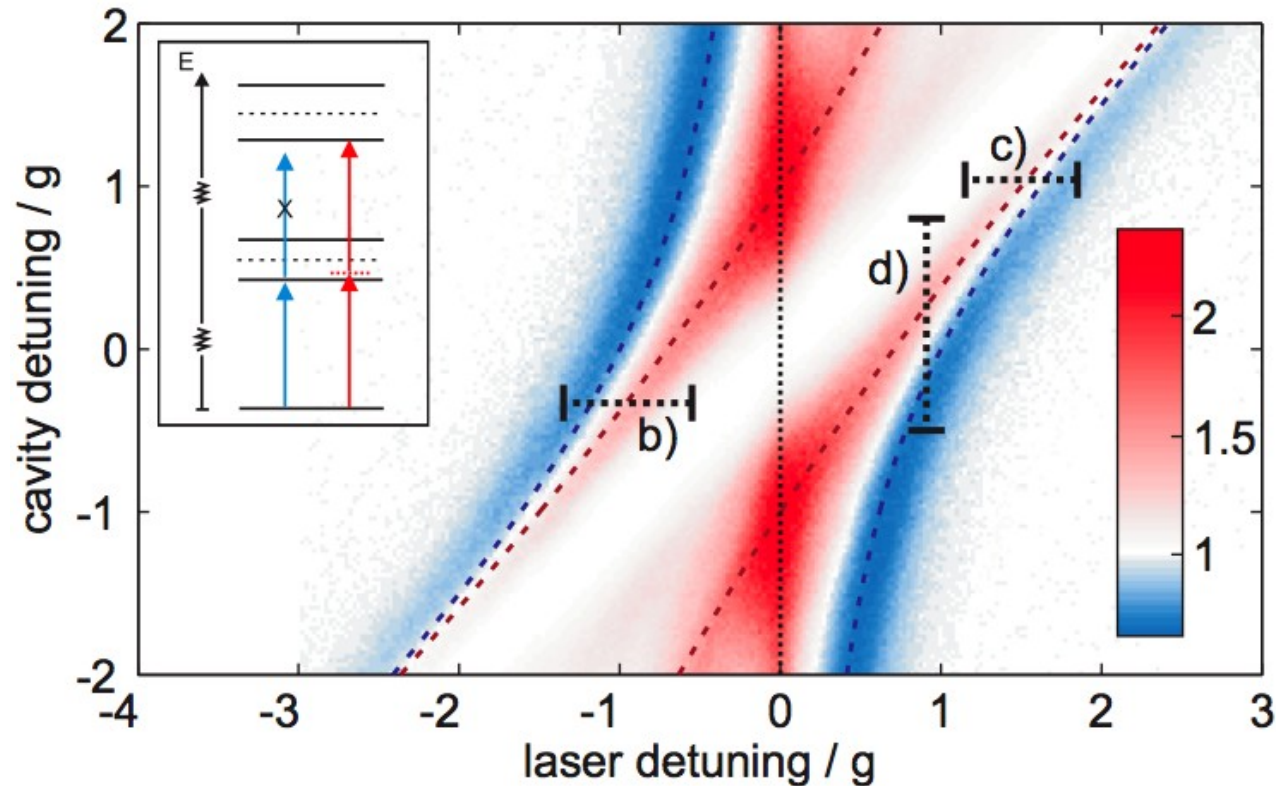


$$\sigma \sim \alpha^4 \frac{\hbar^2}{m^2 c^2} \left(\frac{\hbar \omega}{mc^2} \right)^6$$

Optical bistability and limiting

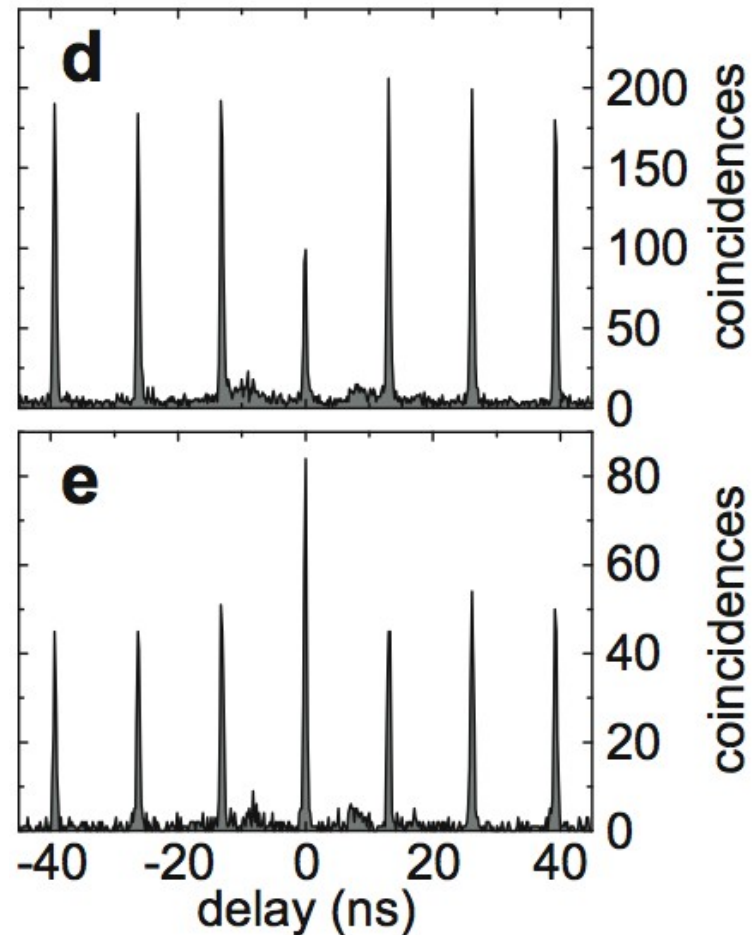


Bunching vs. Antibunching



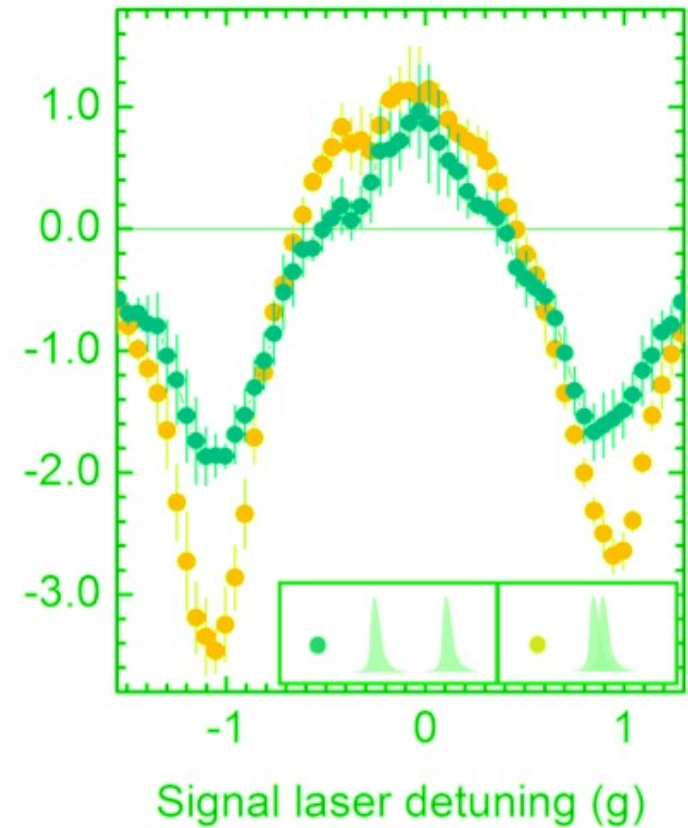
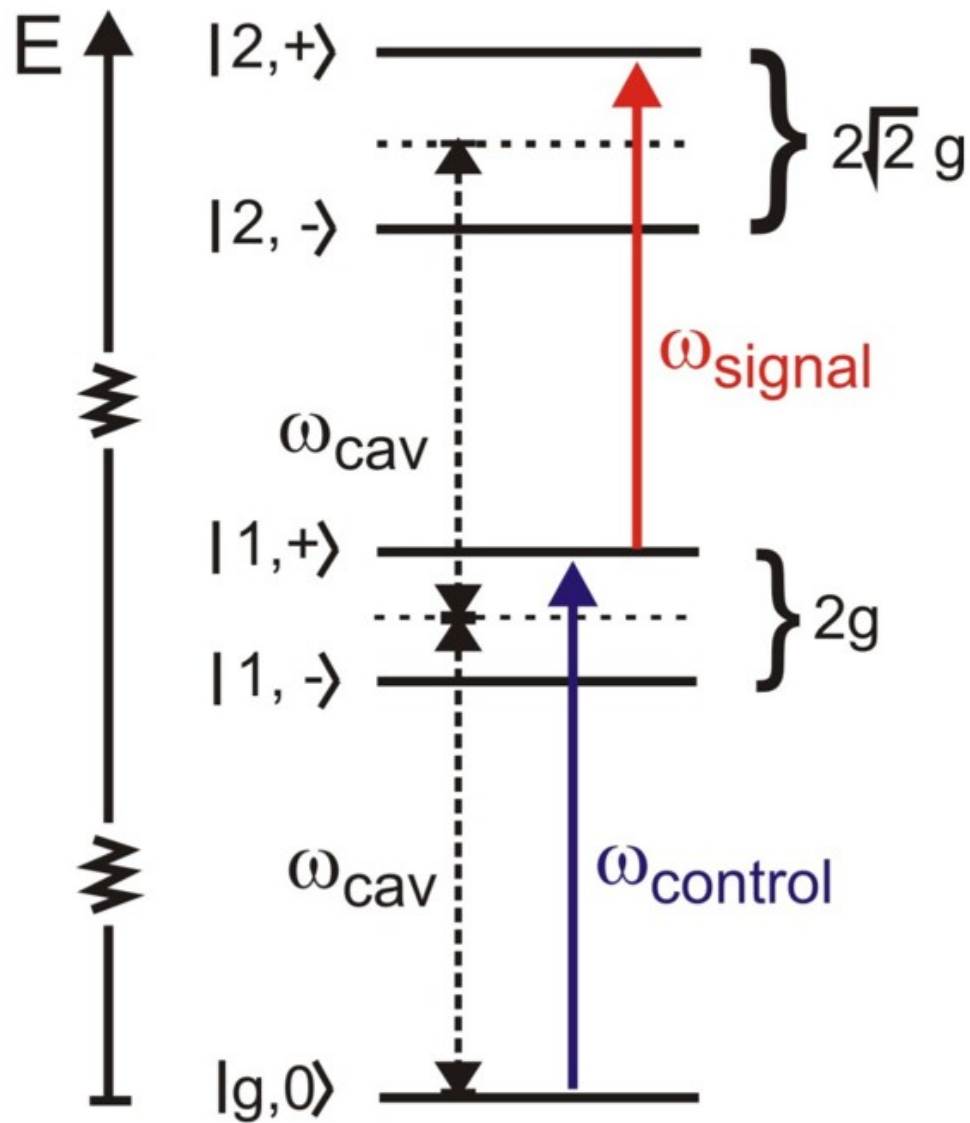
Theoretical picture from: T. Volz et al., *Ultrafast all-optical switching by single photons*, Nature Photonics 6, 605–609 (2012)

Correlation measurement



Experiment from: T. Volz et al., *Ultrafast all-optical switching by single photons*, Nature Photonics 6, 605–609 (2012)

Ultrafast pump & probe

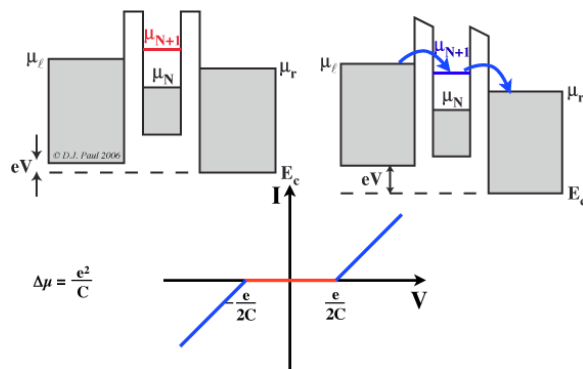


Figures from: A. Reinhard et al., *Strongly correlated photons on a chip*,
Nature Photonics 6, 93-96 (2012)

Photon blockade

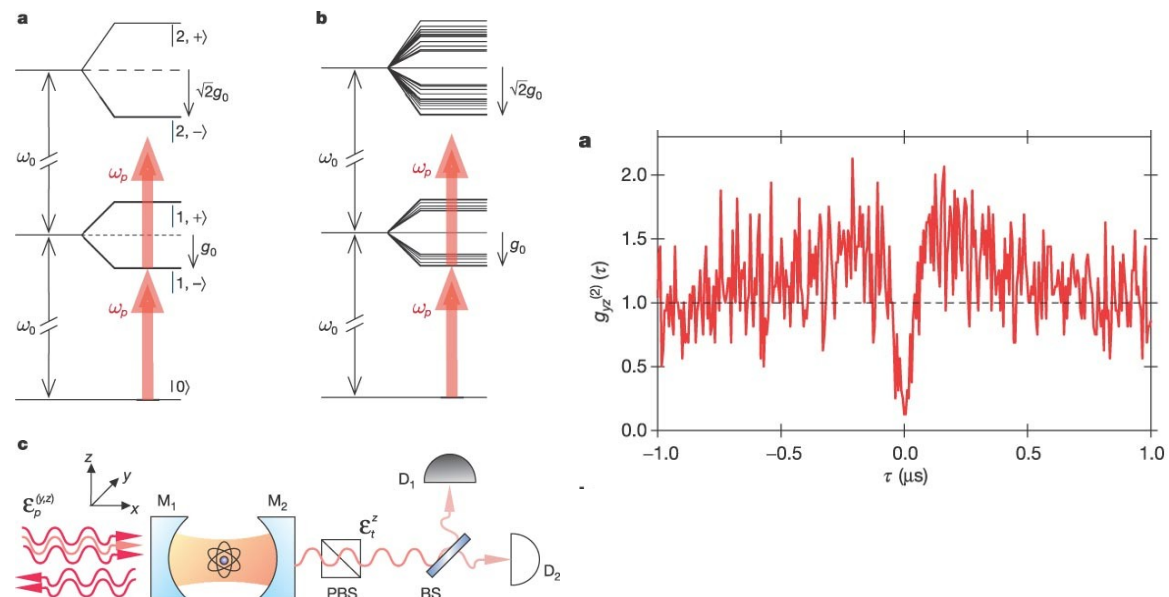
Simplest signature of strong **photon-photon interactions** at **single photon level**

- entrance of **first photon** into cavity **blocks** entrance of a second
- after one photon has exited, system has to **reload**; **dead time** between emitted photons
- **transmitted beam**: **anti-bunched** stream with **sub-Poissonian** statistics
- requires huge $\chi^{(3)}$ **optical nonlinearity**. So far, observed in **single mode cavities** both in visible/IR and in **μ -wave (circuit-QED)** and in bulk **Rydberg atom gases**.
- analog of **Coulomb blockade** of **mesoscopic conductors**



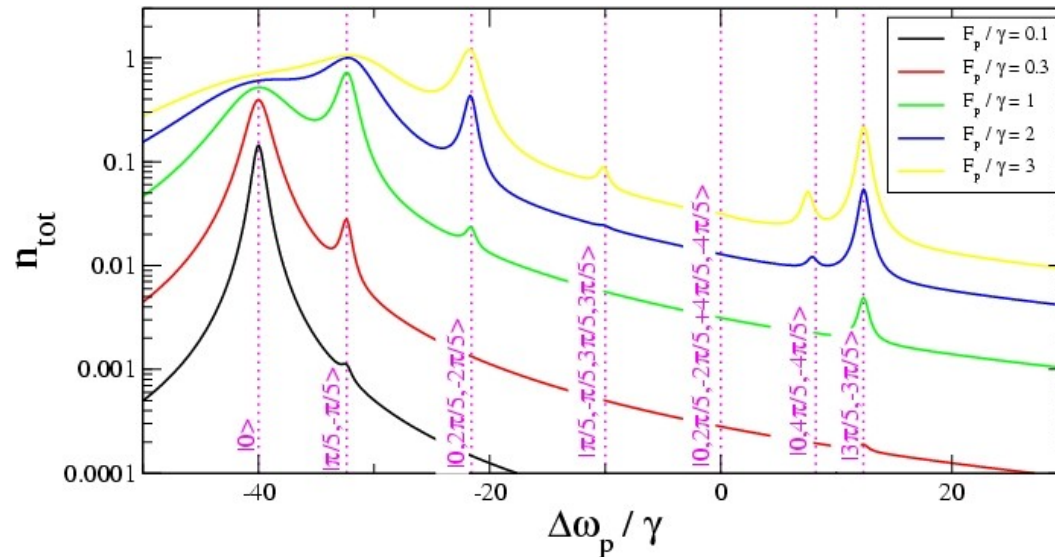
Coulomb blockade

figure D. J. Paul, Cambridge, 2006



from: Birnbaum et al., Nature 436, 87 (2005)

Impenetrable “fermionized” photons in 1D necklaces



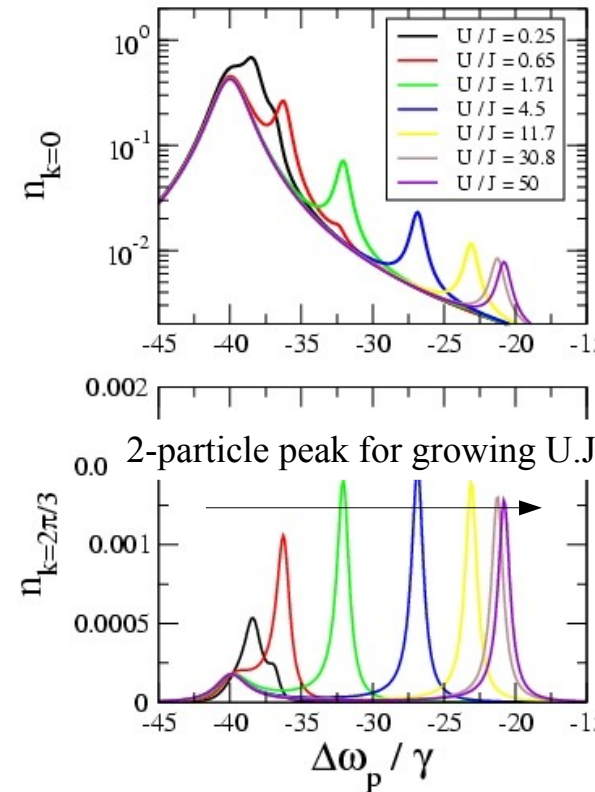
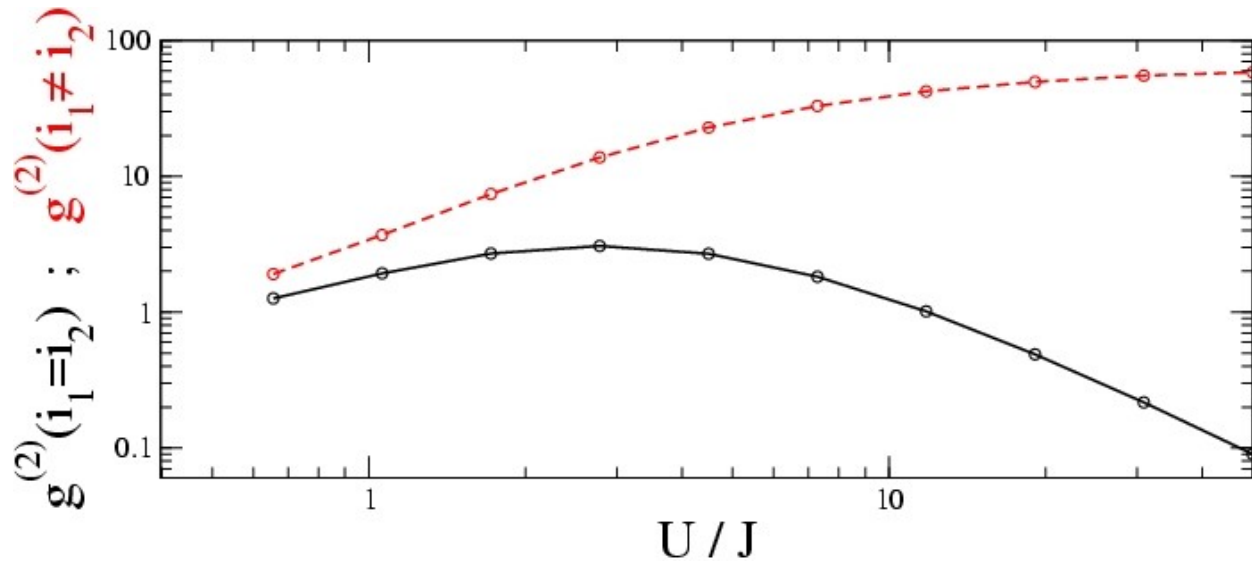
Transmission spectrum as a function pump frequency for fixed pump intensity:

- each peak corresponds to a Tonks-Girardeau many-body state $|q_1, q_2, q_3 \dots \rangle$
- q_i quantized according to PBC/anti-PBC depending on $N=\text{odd/even}$
- $U/J \gg 1$: efficient photon blockade, impenetrable photons.

N-particle state excited by N photon transition:

- Plane wave pump with $k_p=0$: selects states of total momentum $P=0$
- Monochromatic pump at ω_p : resonantly excites states of many-body energy E such that $\omega_p = E / N$

Two-body wavefunction reconstructed from intensity correlations of emission



Finite U/J , pump laser tuned on two-photon resonance

- intensity correlation between the emission from cavities i_1, i_2
- at large U/γ , larger probability of having $N=0$ or 2 photons than $N=1$
 - low $U \ll J$: bunched emission for all pairs of i_1, i_2
 - large $U \gg J$: antibunched emission from a single site
positive correlations between different sites
- Idea straightforwardly extends to more complex many-body states.

If you wish to know more...

REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY–MARCH 2013

Quantum fluids of light

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(2013)

CIRCUMNAVIGATING AN OCEAN OF INCOMPRESSIBLE LIGHT

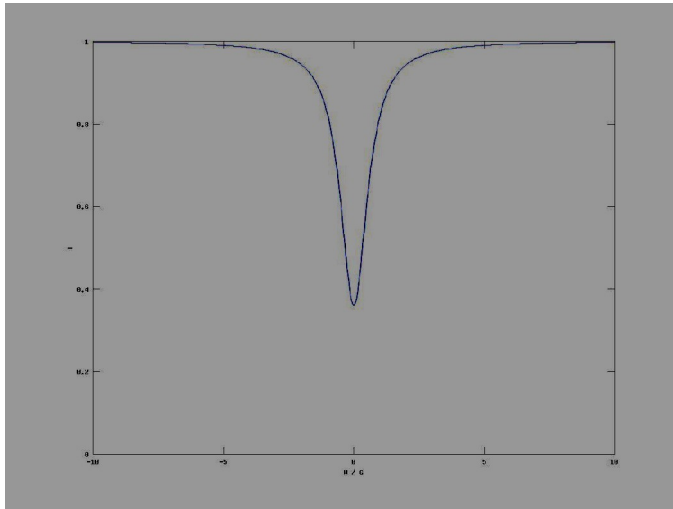
A JOURNEY ACROSS THE EXCITING PERSPECTIVES OF
QUANTUM FLUIDS OF LIGHT

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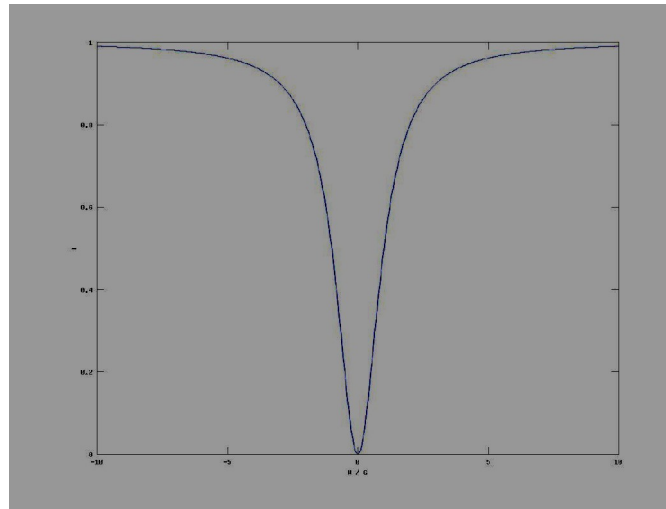
Starting with Newton's breakthrough discovery that the same gravitational force is responsible for apples falling from trees as well as for the Moon orbiting around the Earth, a constant theme in modern physics has been that the same physical mechanism can be active in systems of hugely different size, leading to very diverse observable consequences. Rotation, for instance, is at the root of many observations in astronomical as well as condensed-matter systems, from spiral galaxies to ultra-cold atomic clouds to electron liquids in solids: on an astronomical scale, the arms of spiral galaxies shown in the left panel of fig. 2 originate from a complex interplay of gravity, rotation and star formation in the matter forming the galaxy. On a microscopic scale, the regular arrangement of quantized vortices in a rotating Bose-Einstein condensate shown in the right panel of the same figure is a direct signature of superfluidity of the trapped atomic gas. Given the formal analogy between the Coriolis and the Lorentz force, a most intriguing manifestation of rotation physics in a nanoscopic quantum-mechanical context are the exotic incompressible phases of electron gases in strong magnetic fields with their quantized Hall resistance and the peculiar statistics of their elementary excitations. Inspired by such an interdisciplinary approach, this article will accompany the reader in a journey through the physics of rotating quantum fluids, from Bose-Einstein condensation and superfluidity in ultracold atomic gases, to the most recent perspectives of fractional quantum Hall effects in quantum fluids of light.

Transmission spectra

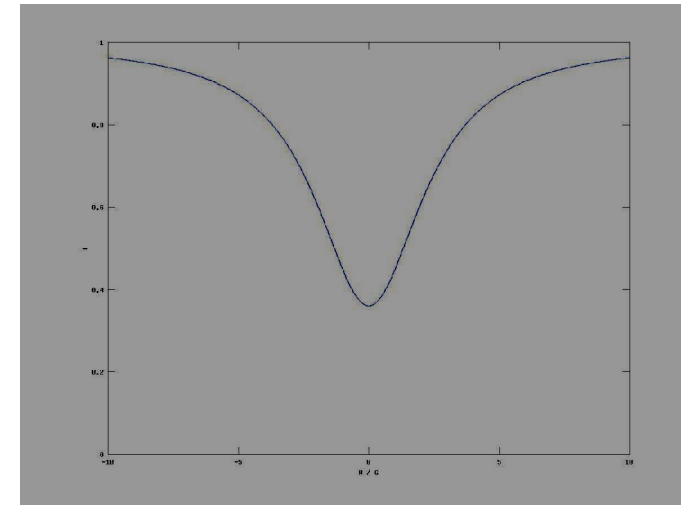


← Undercoupling regime

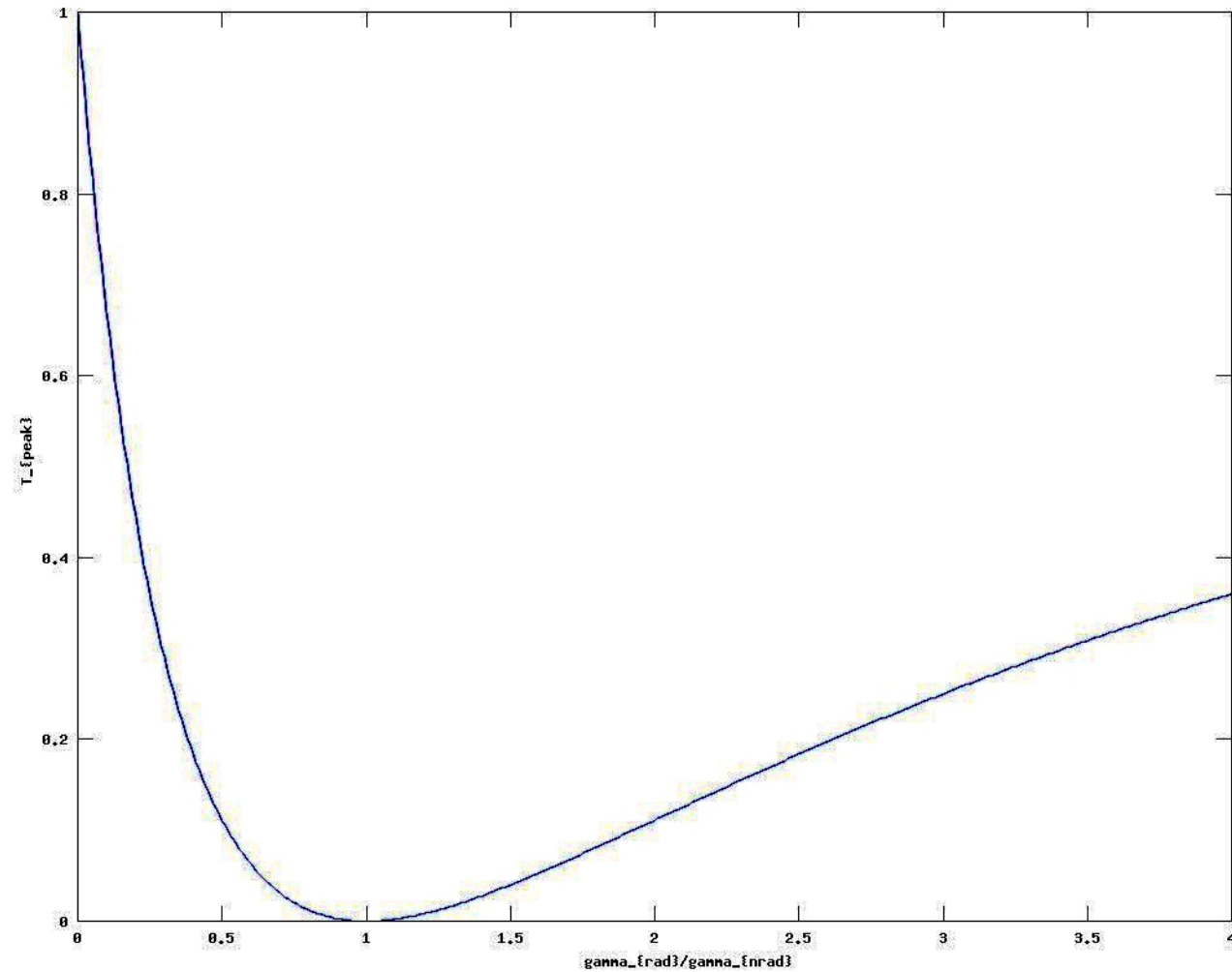
Critical coupling →



Overcoupling regime →



Under vs. over coupling



Fractional quantum Hall effect with light

Bose-Hubbard model
+
synthetic gauge field

$$H_0 = \sum_i \hbar \omega_o \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j e^{i\varphi_{ij}} + \hbar \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

gauge field gives phase in hopping terms

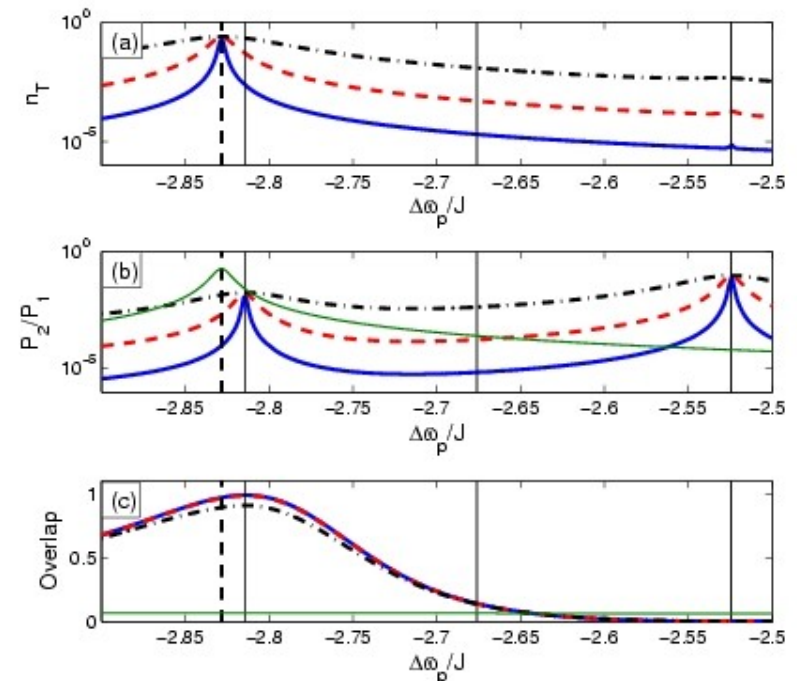
with usual coherent drive and dissipation → look for non-equil. steady state

Transmission spectra:

- peaks correspond to many-body states
- comparison with eigenstates of H_0
- good overlap with Laughlin wf (with PBC)

$$\psi_l(z_1, \dots, z_N) = \mathcal{N}_L F_{\text{CM}}^{(l)}(Z) e^{-\pi \alpha \sum_i y_i^2} \times \prod_{i < j}^N \left(\vartheta \left[\begin{matrix} \frac{1}{2} \\ \frac{1}{2} \end{matrix} \right] \left(\frac{z_i - z_j}{L} \middle| i \right) \right)^2$$

- no need for adiabatic following, etc....

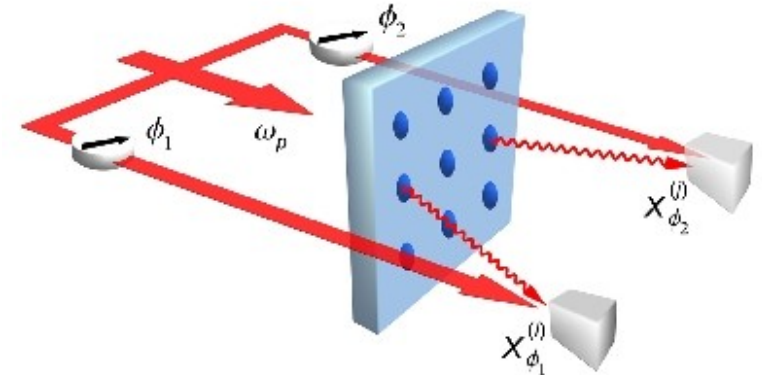


How to directly characterize FQH states?

Homodyne detection of secondary emission

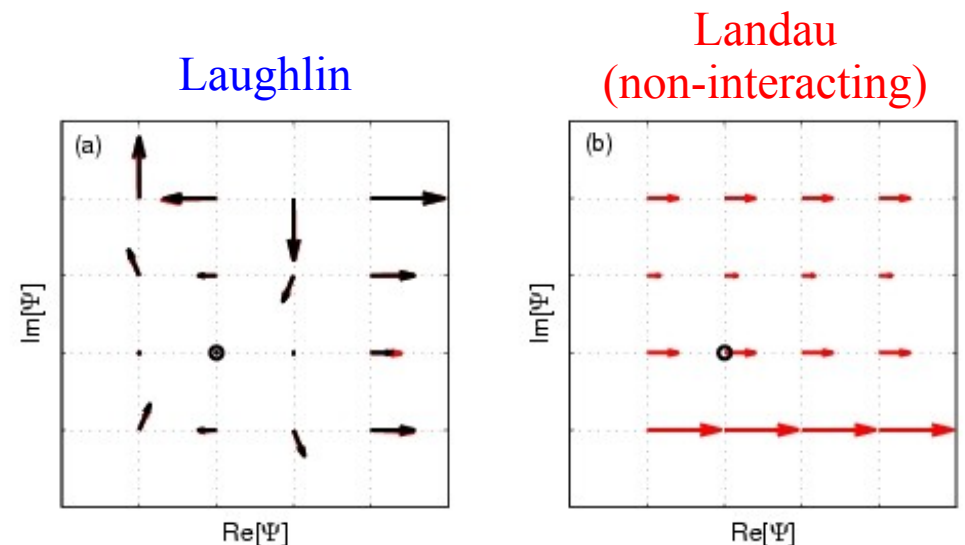
→ info on many-body wavefunction

$$\begin{aligned} \langle \hat{b}_i \hat{b}_j \rangle = & \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle \\ & + i \langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle \end{aligned}$$



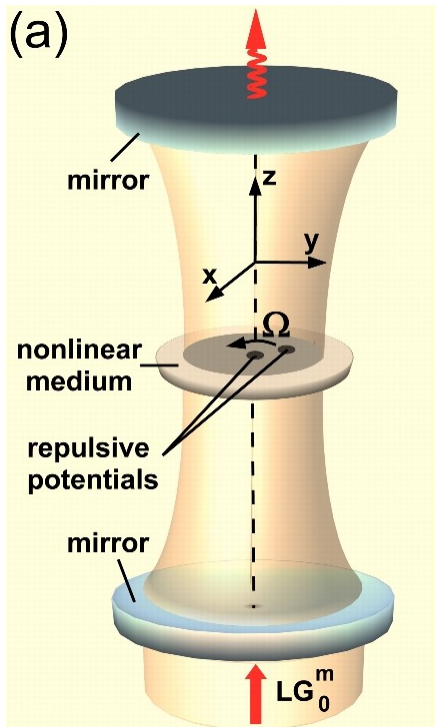
Note: optical signal gauge dependent,
optical phase matters !

Non-trivial structure of Laughlin state
compared to non-interacting photons



Rotating photon fluids

Rotating system at angular speed Ω

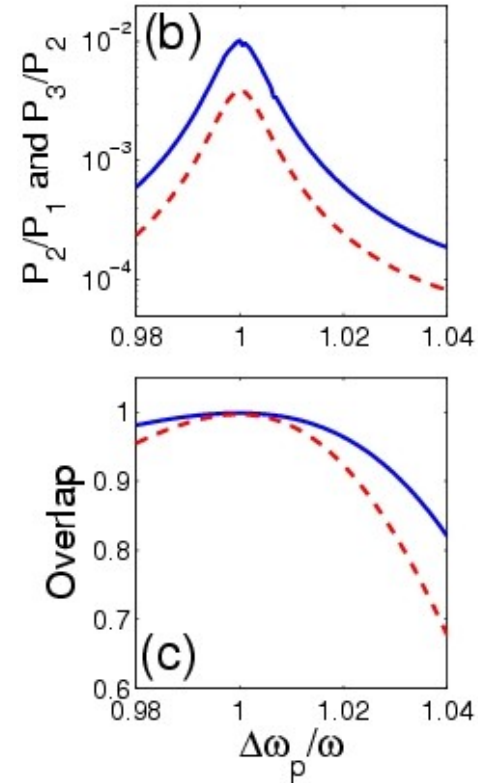


same form \rightarrow Coriolis $F_c = -2m\Omega \times v$
 \rightarrow Lorentz $F_L = e v \times B$

Rotating photon gas injected by LG pump
 with finite orbital angular momentum

Resonant peak in transmission due to Laughlin state:

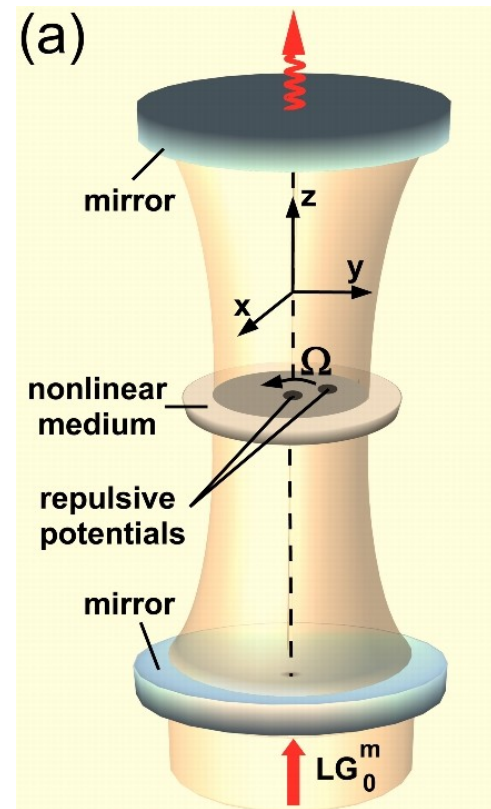
$$\psi(z_1, \dots, z_N) = e^{-\sum_i |z_i|^2/2} \prod_{i < j} (z_i - z_j)^2$$



Overlap measured from quadrature noise of transmitted light

$$\langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle$$

Anyonic braiding phase



- LG pump to create and maintain quantum Hall liquid
- Localized repulsive potentials in trap:
 - create quasi-hole excitation in quantum Hall liquid
 - position of holes adiabatically braided in space
- Anyonic statistics of quasi-hole: many-body Berry phase ϕ_{Br} when positions swapped during braiding
- Berry phase extracted from shift of transmission resonance while repulsive potential moved with period T_{rot} along circle

$$\phi_{Br} \equiv (\Delta\omega_{oo} - \Delta\omega_o) T_{rot} [2\pi]$$

