Supersolidity of excitons

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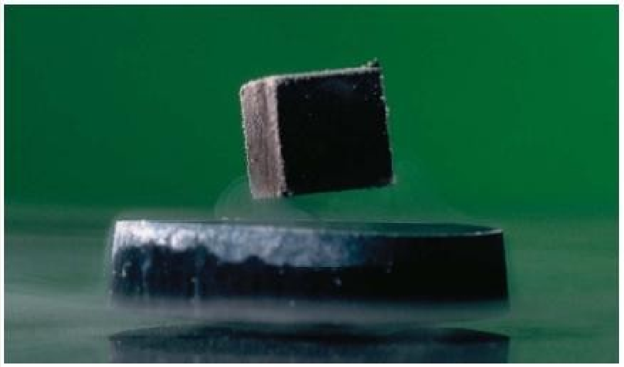
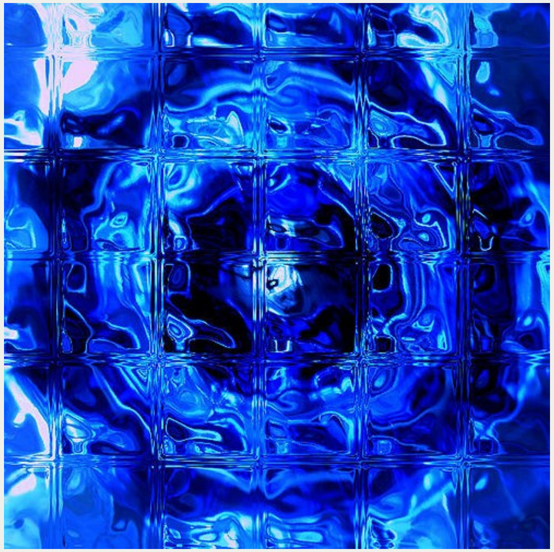
Outline

1. What are supersolids?
2. Search for supersolids in ^{4}He
3. Supersolidity in Bose-Einstein condensates
4. Supersolidity of excitons

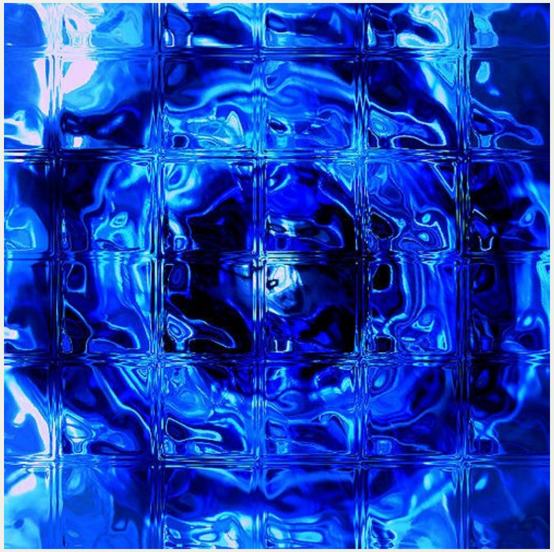
“Superstates” of matter





Superfluidity	Superconductivity	Supersolidity
P. Kapitsa J. F. Allen, D. Misener (1937)	*K. H. Onnes (1911)*	*?*

What are supersolids?



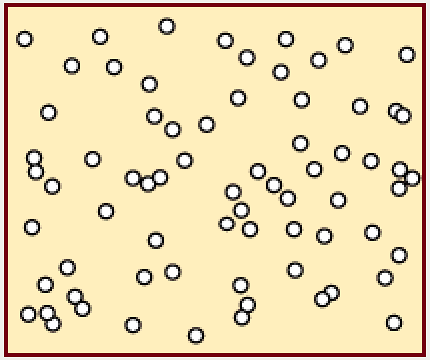
Supersolid = **Superfluid** + **Solid**

Superfluid – persistent flow, zero viscosity, quantized vortices, broken gauge symmetry, Bose-Einstein condensation

Solid – periodic order, atom localization, resistance to shear

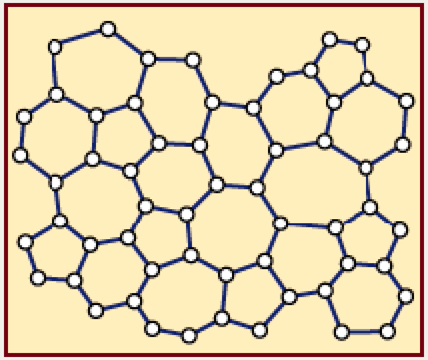
Is it possible to realize both these, seemingly contradictory properties in a single quantum phase?

Order in solids



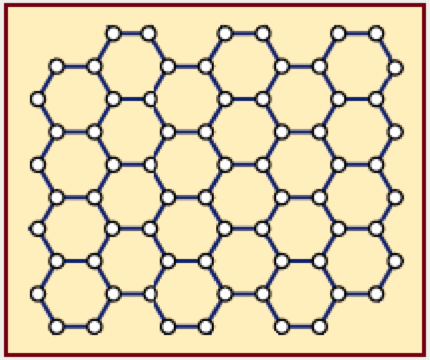
Gas or Liquid

Translational invariance $\delta\rho(\boldsymbol{r}) = \langle \rho(\boldsymbol{r},t) - \bar{\rho} \rangle_t = 0$



Amorphous solid

Broken translational invariance $\delta\rho(\boldsymbol{r}) \neq 0$



Crystal

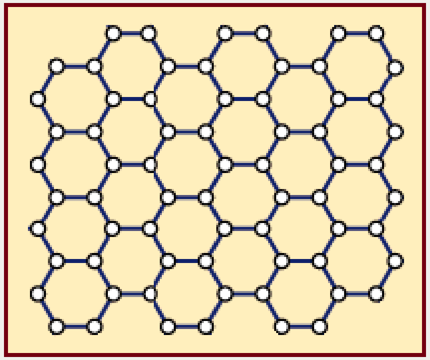
Density long range order $\delta\rho(\boldsymbol{r}) = \delta\rho(\boldsymbol{r}+\boldsymbol{T})$

Order in solids

Atoms are pretty much fixed in their positions in the lattice

No flow, resistance to shear

The long range order appears in solids **spontaneously**, it is not due to any external forces



Crystal

Density long range order $\delta\rho(\boldsymbol{r}) = \delta\rho(\boldsymbol{r}+\boldsymbol{T})$

Order in a superfluid

In general, the system can be split into the superfluid and normal component

$$\rho(\boldsymbol{r}) = \rho_S(\boldsymbol{r}) + \rho_N(\boldsymbol{r})$$

The superfluid component carries no entropy and flows without dissipation below the critical velocity.

Superfluidity is a manifestation of quantum particles behaving collectively as some complex field ψ. For example, quantum vortex is a defect of its phase.

This kind of description appears at large occupation numbers, which can be realized in the case of Bose statistics (condensation). In the case of fermions, superfluidity may occur if the particles form pairs which act like bosons (eg. in superconductors).

Order in a superfluid

To illustrate the order of a superfluid, we consider the one-particle density matrix

$$n(\mathbf{r}, \mathbf{r}') = \langle \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}') \rangle$$

or its average

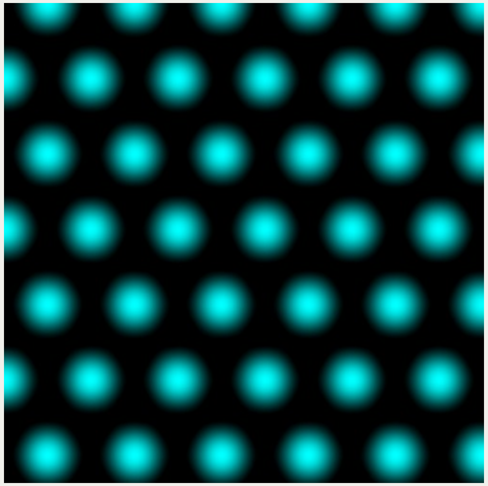
$$n(\mathbf{r}) = \frac{1}{\Omega} \int d^3r' \, n(\mathbf{r}', \mathbf{r}' + \mathbf{r})$$

In a superfluid, $n(\mathbf{r})$ does not decrease to zero (or decreases slowly) as $|\mathbf{r}|$ increases to infinity. This so called off-diagonal long range order (ODLRO) means that quantum correlations between distant positions are present.

This behavior can be understood in that the particles are substantially **delocalized** through the system.

How to reconcile this with **localization** of particles in solids?

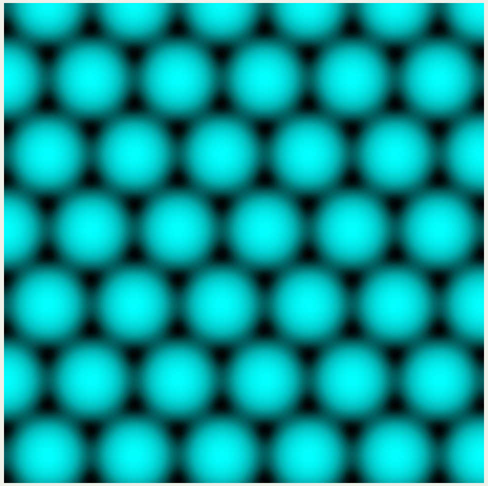
Are supersolids at all possible?



O. Penrose and L. Onsager (1956)
atoms localized
no exchanges of particles

$$\Psi(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_N)=\prod_i \psi_i(\boldsymbol{r}_i)$$

No superfluidity possible



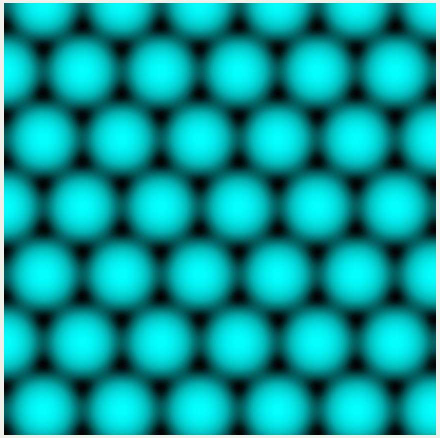
E. Gross (1957)
complex field,
complete delocalization

$$\Psi(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_N)=\prod_i \psi_0(\boldsymbol{r}_i)$$

Superfluidity present

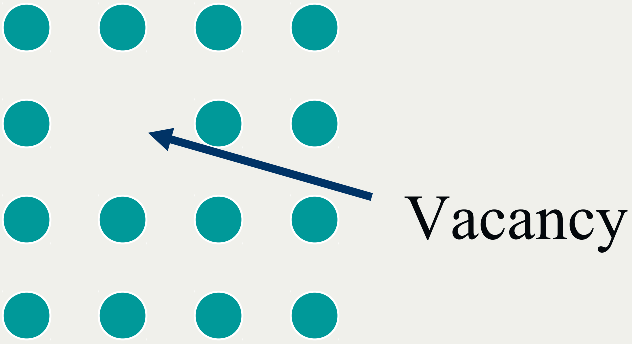
Different kinds of supersolids

1.



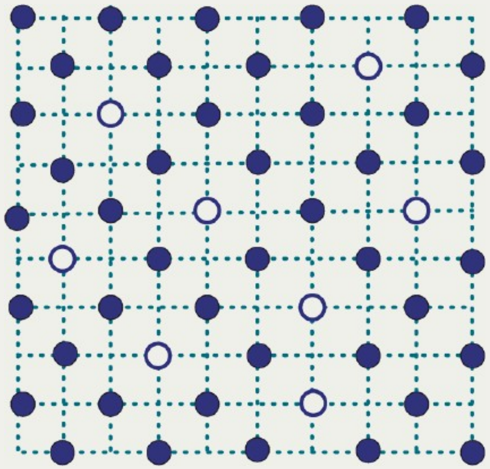
E. Gross (1957)
complete delocalization

2.



A. F. Andreev, I. M. Lifshitz (1969)
Superfluidity of vacancies

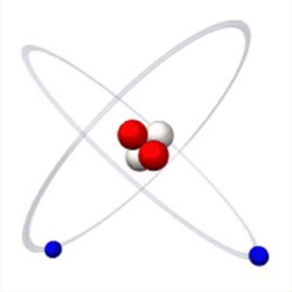
3.



Lattice supersolids

Helium-4

Helium-4 has always been considered as the best candidate for observation of supersolidity due to its unique properties



- Superfluidity observed below few Kelvin
- Turns into solid under moderate pressure (~25 atm)
- High level of delocalization due to the light mass and weakness of the interatomic potential

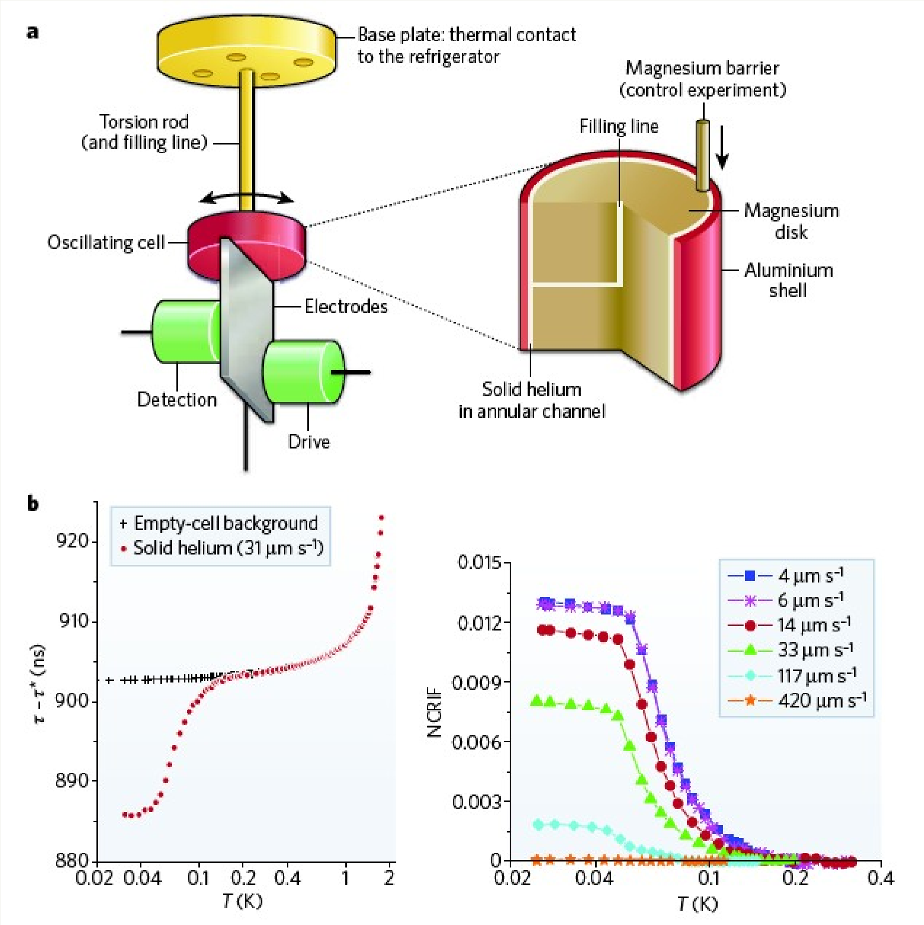
Torsional oscillator experiment

In 1970, A. Leggett predicted that supersolidity would manifest itself in Non-Classical Rotational Inertia

Only the normal part would contribute to oscillations of a supersolid cyllinder

$$\rho(\boldsymbol{r}) = \rho_S(\boldsymbol{r}) + \rho_N(\boldsymbol{r})$$

The moment of inertia is reduced, the oscillations are faster



E. Kim and M. Chan (2004)

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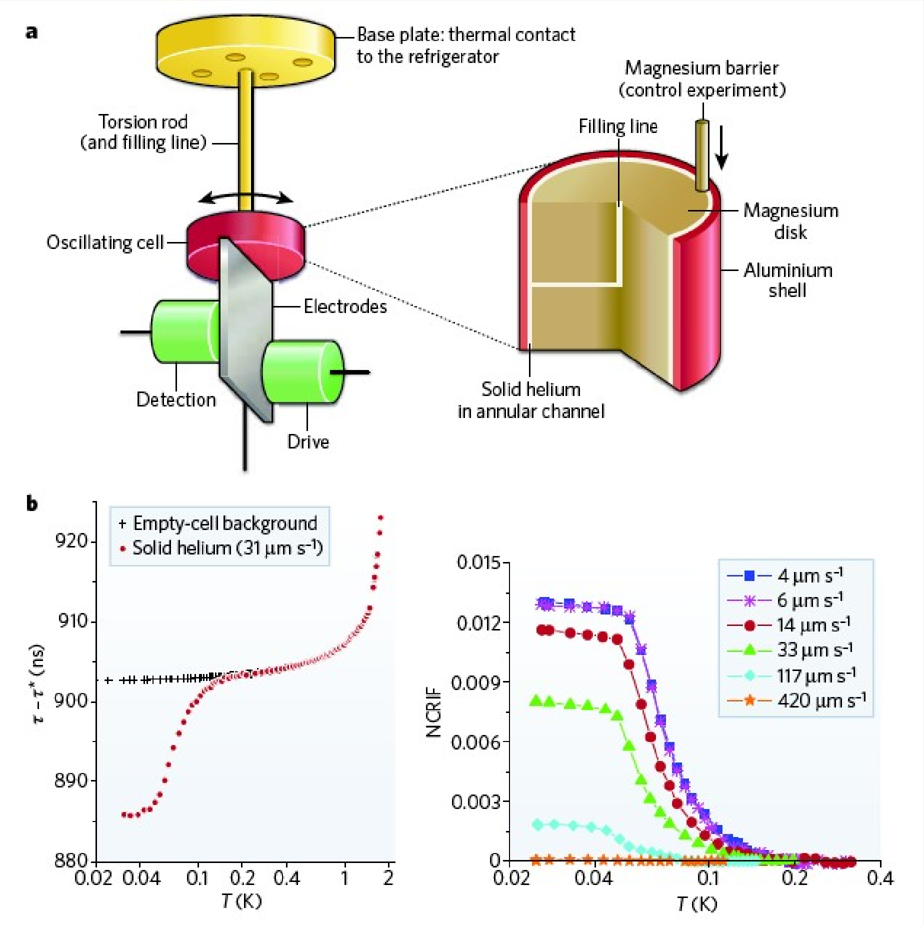
Control experiments used a magnesium barrier or ^{3}He

The effect is suppressed above the critical velocity

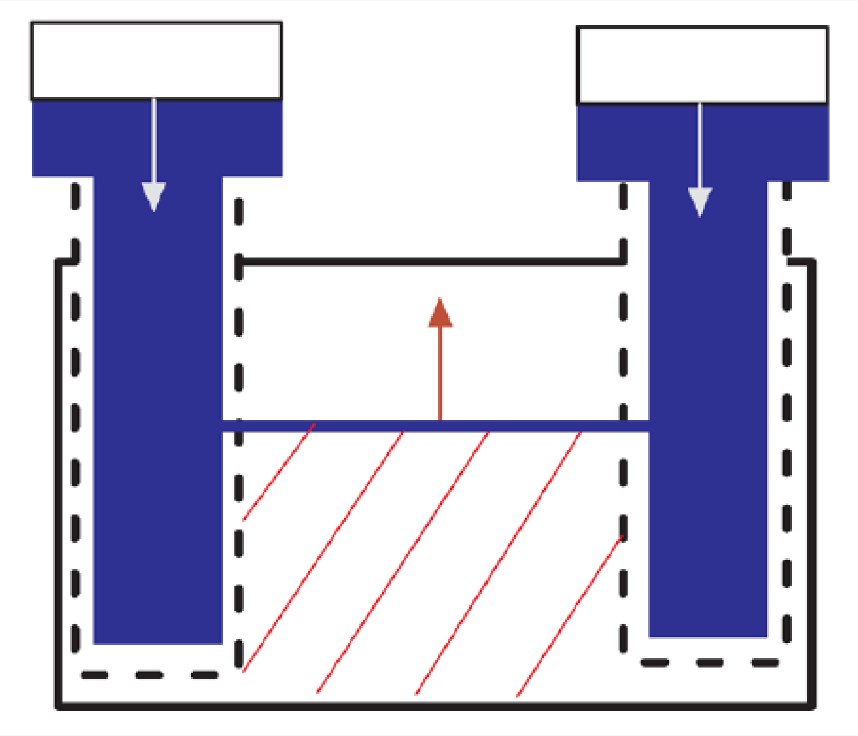


E. Kim and M. Chan (2004)

Superclimb effect

Supersolid should allow for the transport of particles through quantum exchanges of particles, but only particles of the same substance

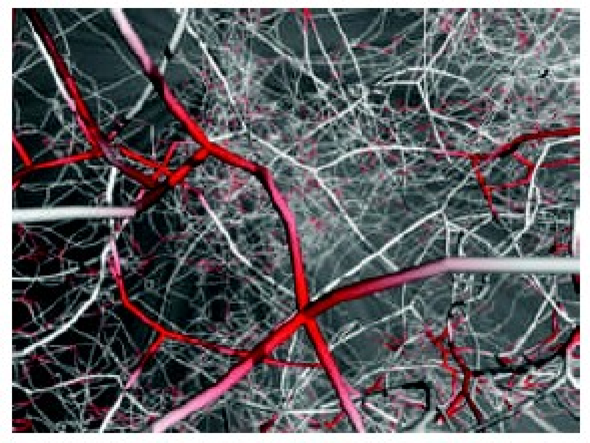
In the experiment, liquid helium was fed from the reservoirs to build up extra plane of atoms



M. W. Ray and R. B. Hallock (2009)

Problems with interpretation

- The supersolid signal in torsional oscillator experiment is much weaker if solid helium crystal is heated to remove imperfections before being cooled down again, the opposite of what the classic theory would predict. Possible interpretation – superflow only along grain boundaries



- The stiffness of helium-4 increases at low temperatures in a way that may mimick the signal attributed to the supersolid (because a stiffer crystal would oscillate more quickly)

Is it really supersolidity?

"I believe that supersolidity [of ^4He] is a real phenomenon but that it is not yet understood."

S. Balibar (2010)

"Consensus is still lacking, as to whether the wealth of experimental evidence accumulated for solid helium indisputably points to the observation of the supersolid phase of matter."

"Regardless of how the current controversy over the interpretation of the present ^4He experiments is eventually resolved, it seems fair to state that solid helium does not afford a direct, simple, and clear observation of the supersolid phenomenon."

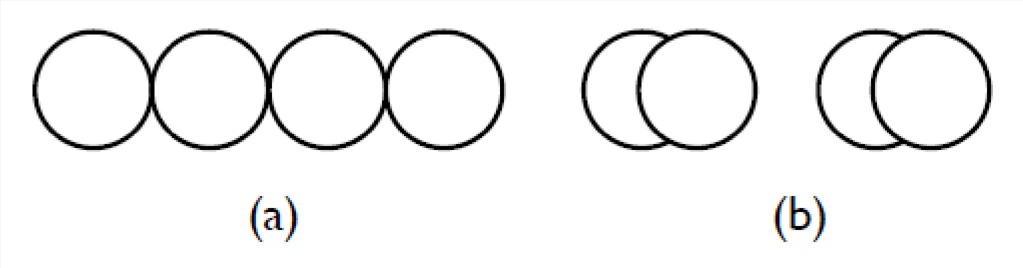
M. Boninsegni and N. V. Prokof'ev (2012)

Are there better candidates for superfluidity?

What kinds of interaction potentials support supersolidity?

Consider particles interacting through a soft sphere potential

$$V(r) = \begin{cases} V_0 & for\, r \leq a \\ 0 & for\, r > a \end{cases}$$

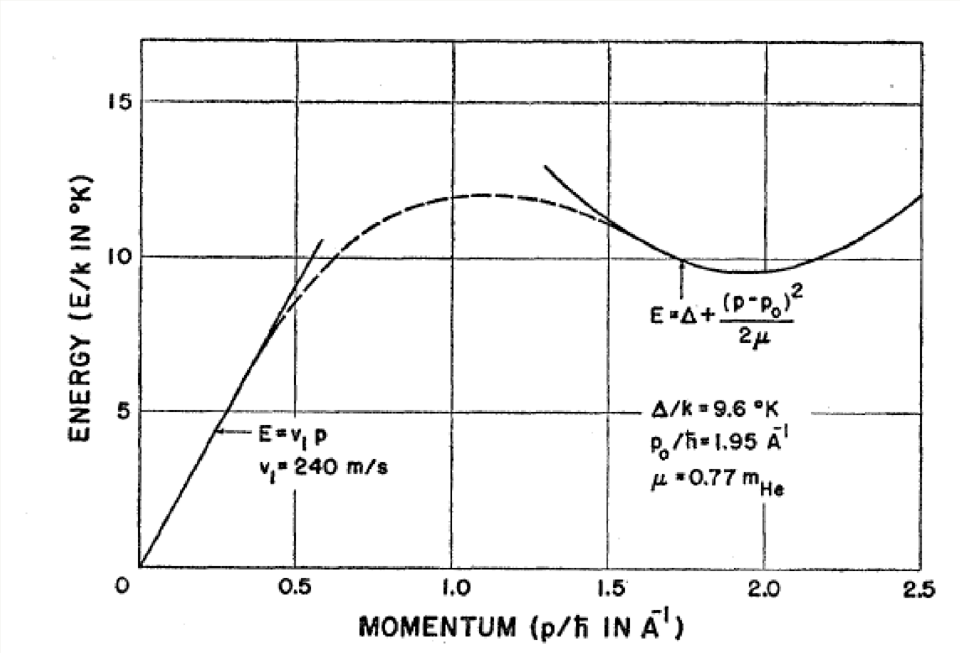


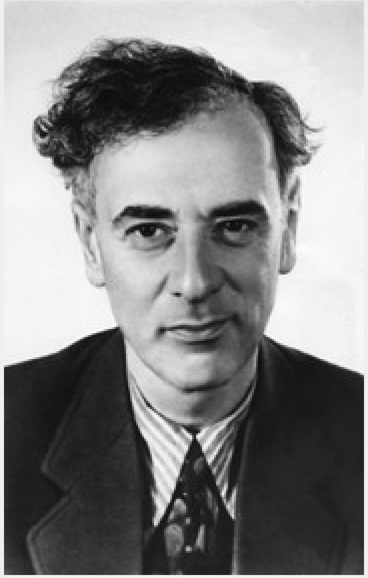
If average distance is slightly less than *a*, configuration (a) costs twice as much energy per particle than configuration (b). It is favourable for the particles to clump together at certain spots.

In a supersolid, probability of finding a delocalized particle at these spots is high.

What about more complicated potentials?

The existence of a **roton minimum** in the spectrum of a system may suggest the existence of a supersolid.





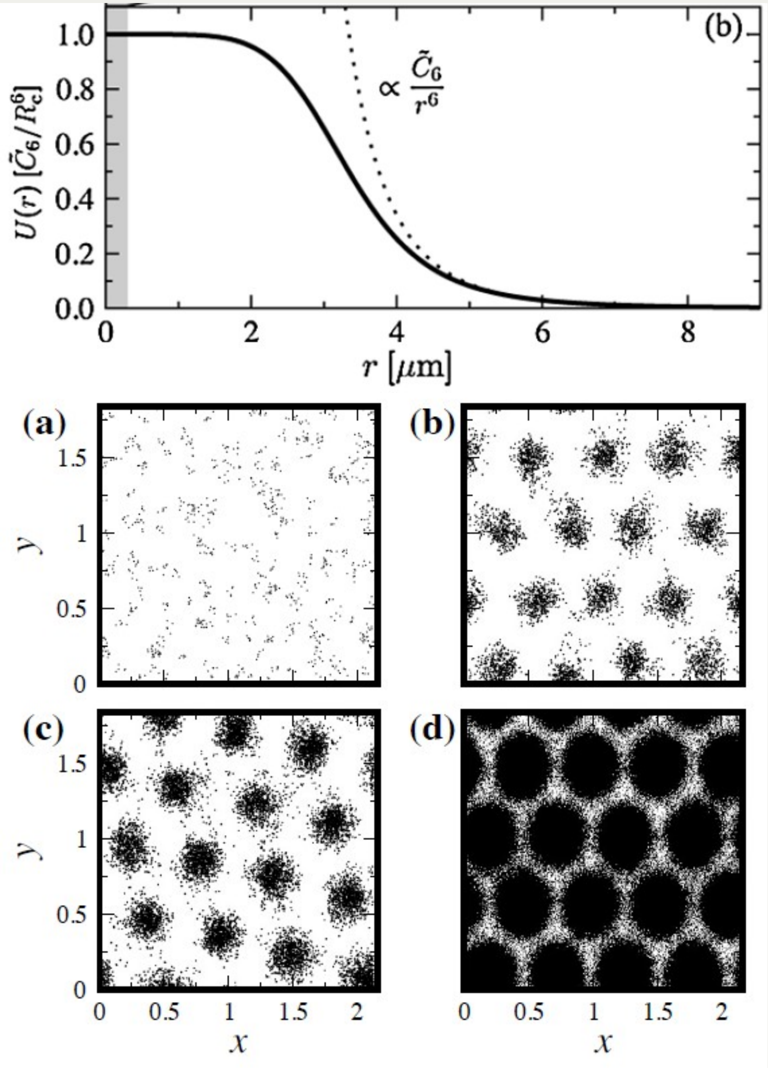
If the roton minimum falls below zero energy, the system becomes unstable against fluctuations of certain wavelength, related to the supersolid lattice constant

Supersolids of cold atoms

In a cold atomic gas, it is possible to prepare a soft-sphere-like interaction between Rydberg-excited atoms. It could be used to achieve a supersolid in the regime of Bose condensation

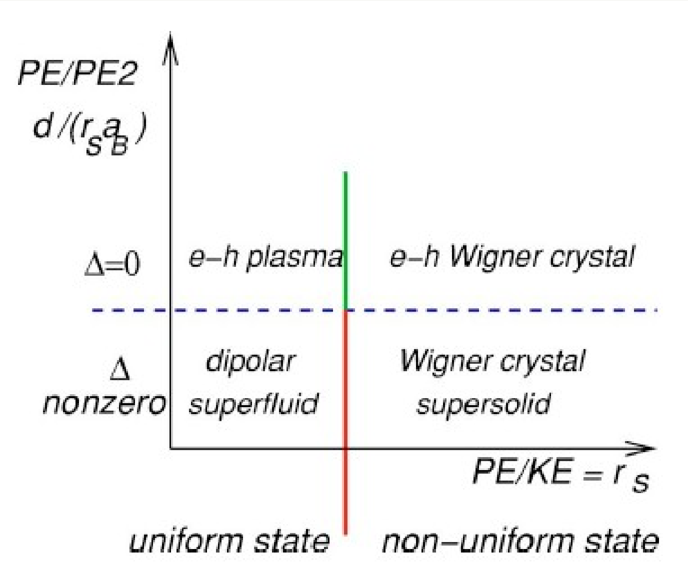
This has been confirmed by both the mean field and Quantum Monte Carlo simulations.

N. Henkel, R. Nath, and T. Pohl, (2010)
F. Cinti et al., (2010)



Supersolidity of excitons

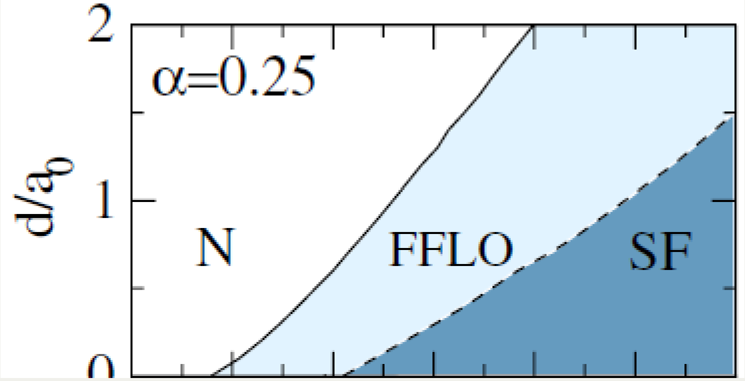
“Wigner supersolid” of excitons in electron-hole bilayers
Y. N. Joglekar, A. V. Balatsky, and S. Das Sarma (2006)



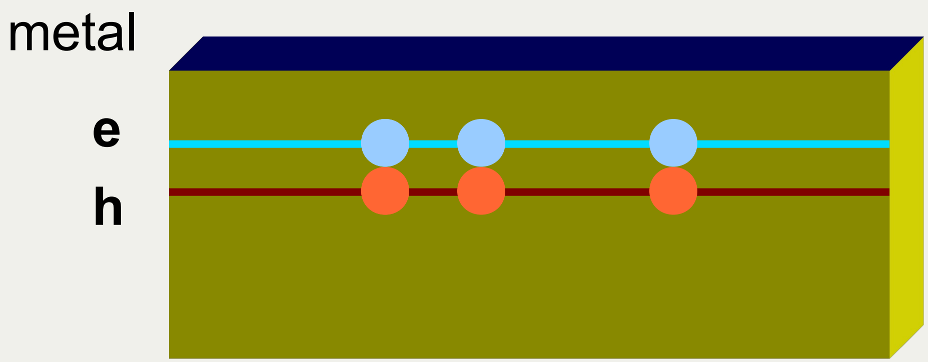
Fulde-Ferrell-Larkin-Ovchinnikov phase (with finite k) in extremely imbalanced bilayers

Minority particles may condense to a supersolid

M. M. Parish, F. M. Marchetti and P. B. Littlewood (2011)



Hybrid Bose-Fermi systems



- The presence of free electron gas in the metallic layer modifies the dipolar interaction between excitons

- The effective exciton-exciton interaction potential is calculated within the Random Phase Approximation. We also take into account the electron-hole pair correlations

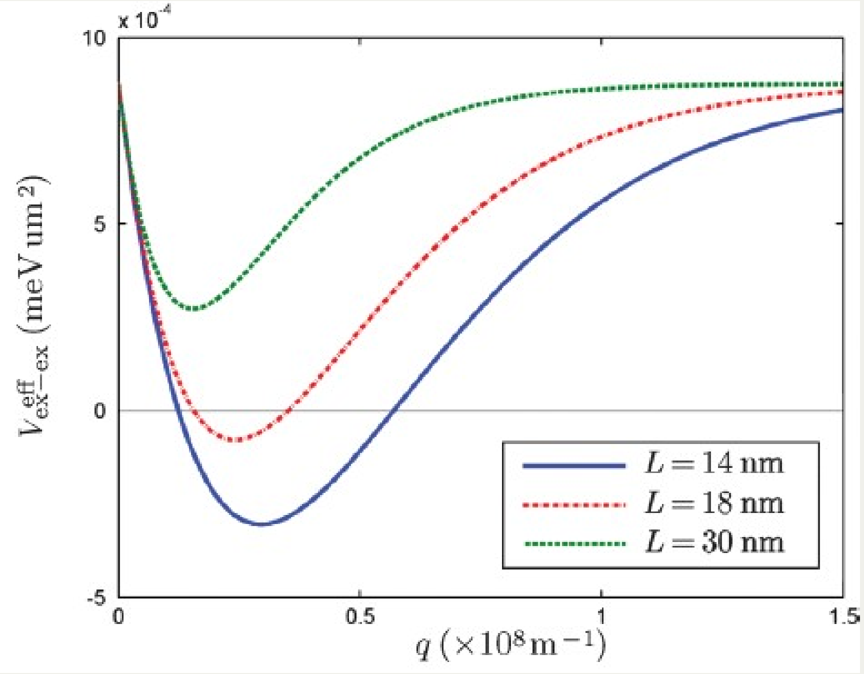
- The exciton condensate is described by the Hartree-Fock Ansatz

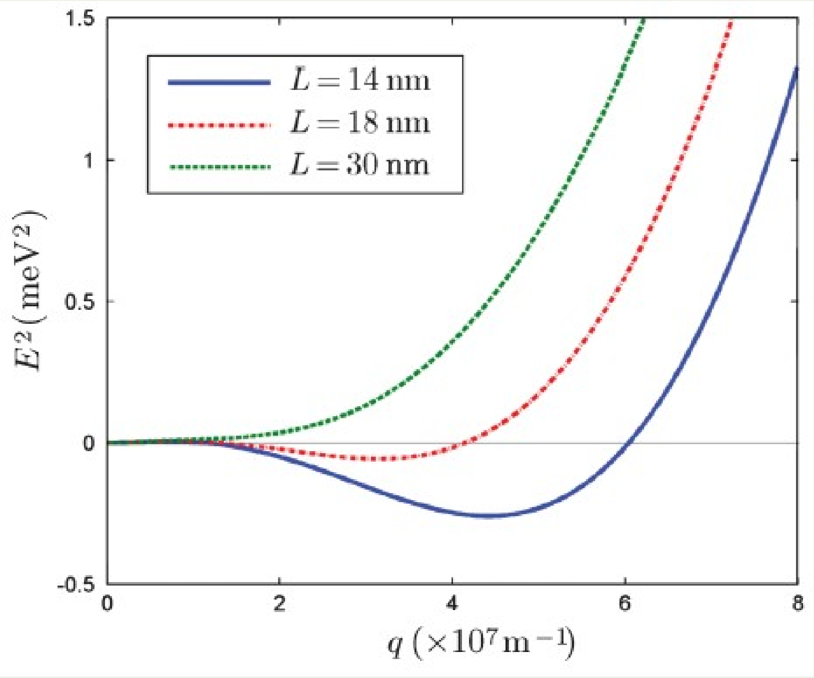
$$\Psi_{\pm}(\mathbf{r}_1, \ldots, \mathbf{r}_N, t) = \prod_i \psi_{\pm}(\mathbf{r}_i, t)$$

Interaction potential and spectra

$$\Psi_{\pm}(\mathbf{r}_1, \ldots, \mathbf{r}_N, t) = \prod_i \psi_{\pm}(\mathbf{r}_i, t)$$

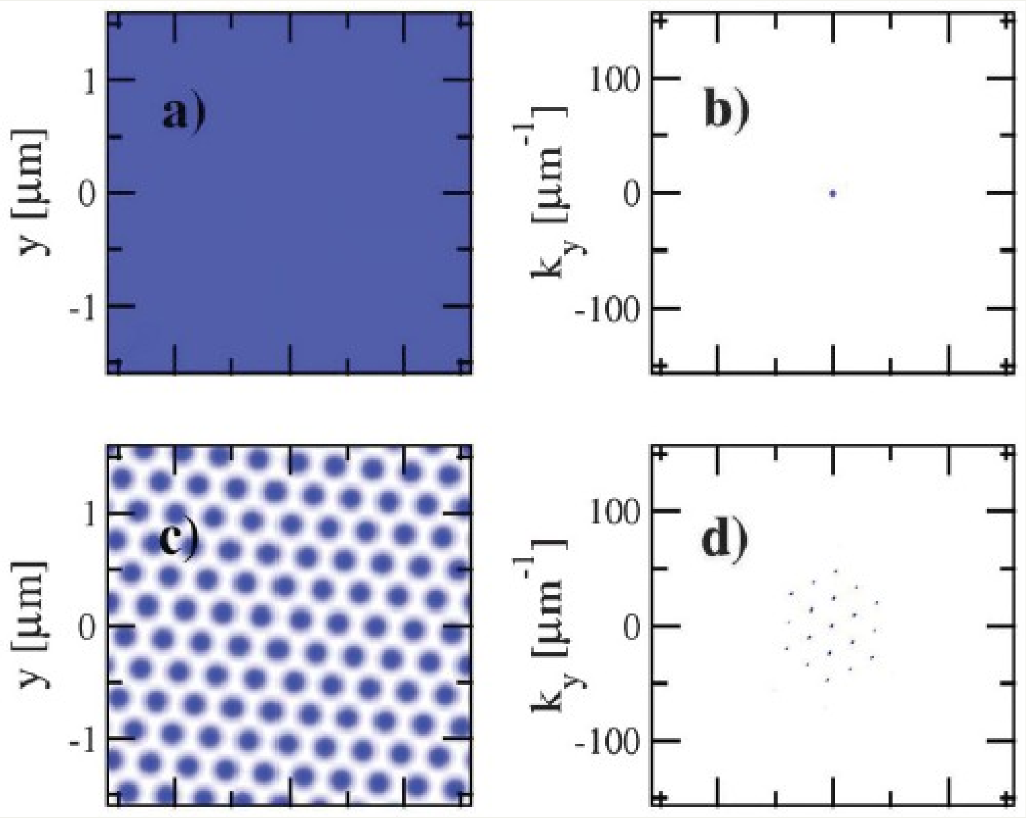
$$i\hbar \frac{\partial \psi_{\pm}(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2 \nabla^2}{2m_{\text{ex}}} \psi_{\pm}(\mathbf{r}, t) + \int V^{\text{eff}}_{\text{ex-ex}}(\mathbf{r} - \mathbf{r}') n_{\text{tot}}(\mathbf{r}', t) d\mathbf{r}' \psi_{\pm}(\mathbf{r}, t) + \alpha n_{\mp}(\mathbf{r}, t) \psi_{\pm}(\mathbf{r}, t).$$





I. Shelykh, T. Taylor, and A. V. Kavokin (2010)

Numerical results



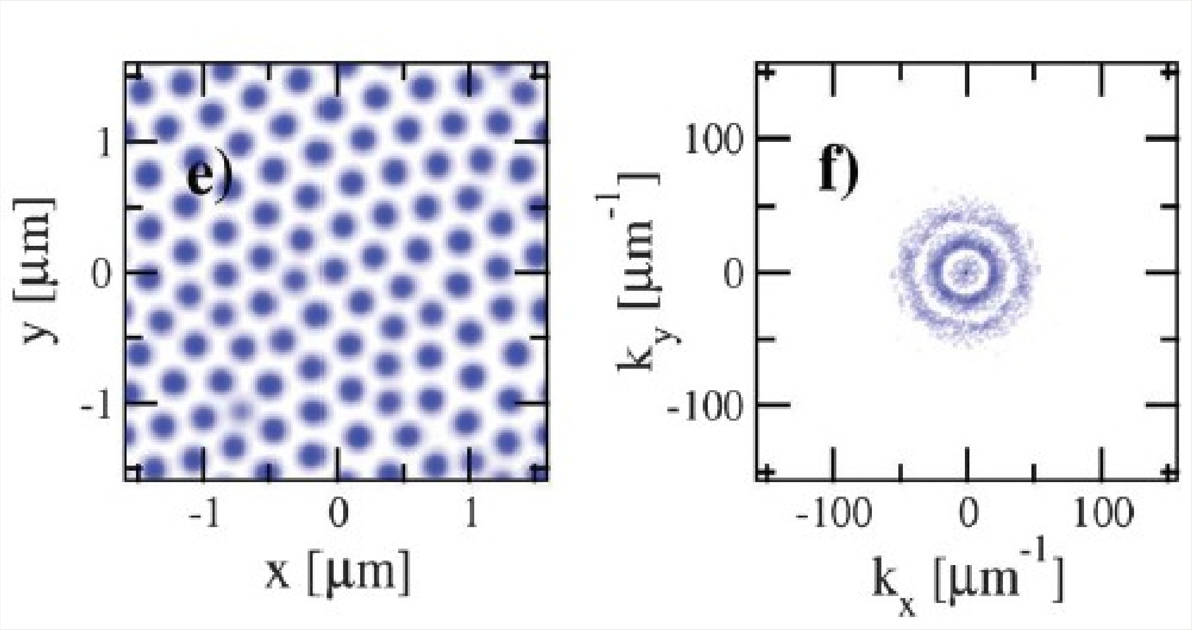
M. Matuszewski, T. Taylor, A.V. Kavokin, PRL **108**, 060401 (2012)

At high enough exciton concentrations, the transition to the supersolid phase is predicted, visible in both real and reciprocal space.

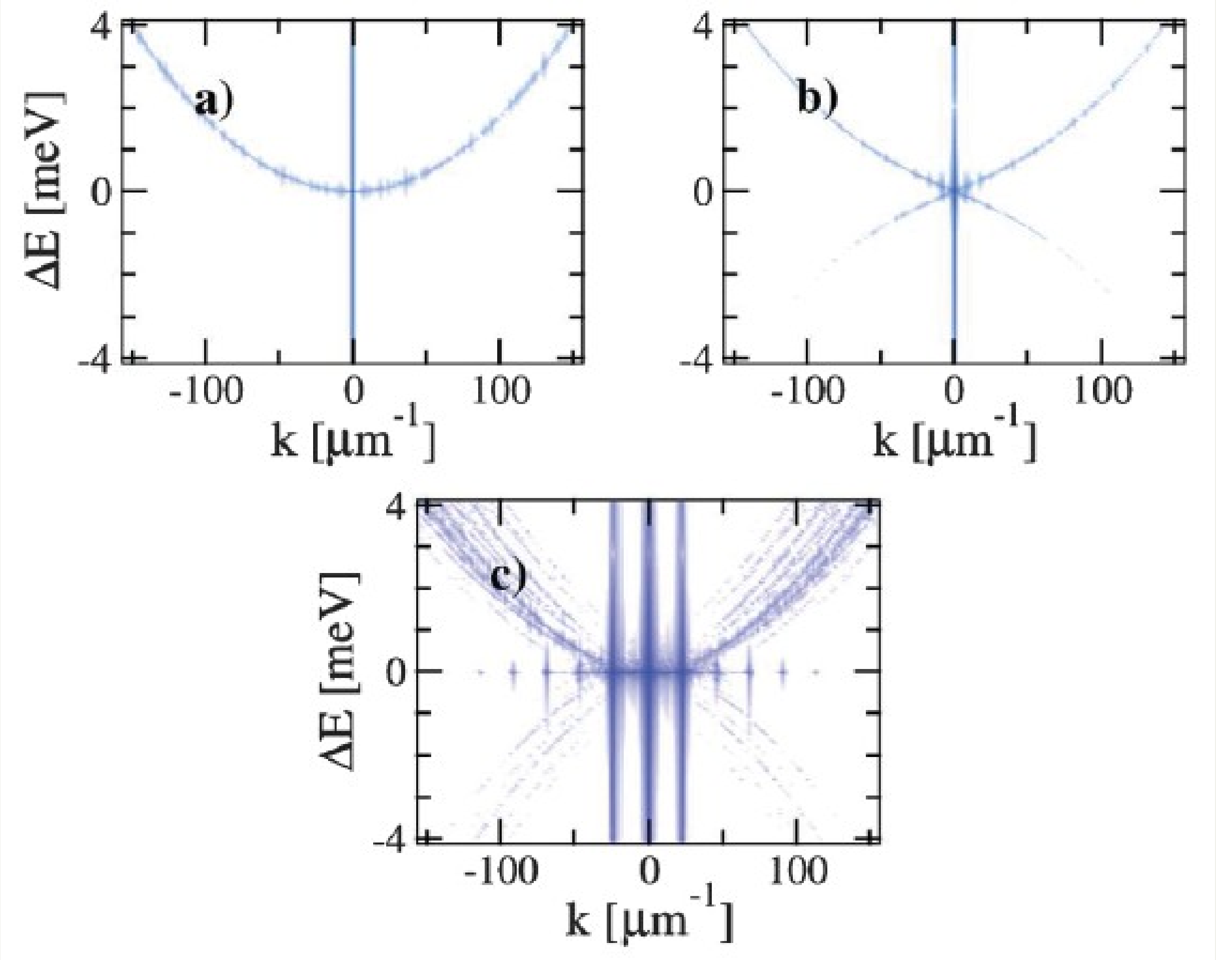
Appropriate design of the structure is necessary (eg. distance between the layers)

Numerical results

The minimization can also find local energy minima, corresponding to lattices with defects

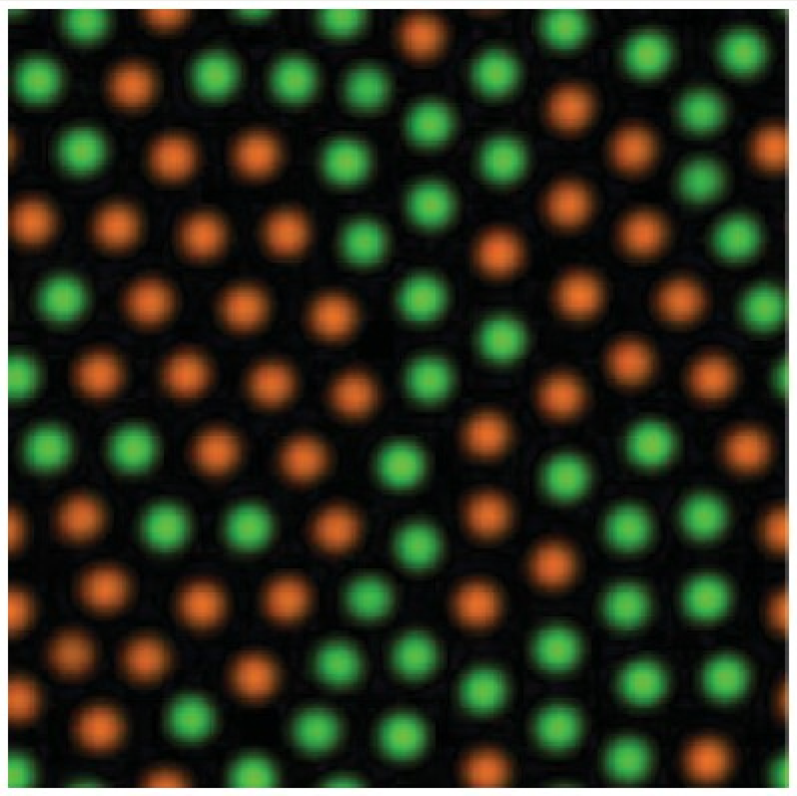


In momentum space, such configurations correspond to concentric rings



The transition is also apparent in the excitation spectrum of the condensate.
With increasing density, we observe the transitions :

free particle spectrum → Bogoliubov spectrum → supersolid spectrum.



In the spin-dependent case, we predict the formation of ferromagnetic spin domains consisting many lattice sites.

Summary

- Supersolid phases, possessing both superfluid and solid long range orders, are the topic of intensive research

- The microscopic mechanisms of supersolidity in solid ^{4}He and its very existence are still under debate

- Bose-Einstein condensates with adjustable interaction potentials are a promising alternative

- Supersolidity in a hybrid system of excitons in presence of an electron gas is predicted to display supersolidity in certain conditions

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